EXPECTED VALUE ANALYSIS OF THE CENTER FOR NAVAL ANALYSES COMPUTER WAR GAME — SEALIFT

WALTER J. KIRSCH

38

EXPECTED VALUE ANALYSIS OF

THE CENTER FOR NAVAL ANALYSES

COMPUTER WAR GAME

- SEALIFT -

* * * * *

Walter J. Kirsch III

EXPECTED VALUE ANALYSIS OF

THE CENTER FOR NAVAL ANALYSES

COMPUTER WAR GAME

- SEALIFT -

by

Walter J. Kirsch III

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
OPERATIONS RESEARCH

United States Naval Postgraduate School
Monterey, California

1 9 6 5

EXPECTED VALUE ANALYSIS OF

THE CENTER FOR NAVAL ANALYSES

COMPUTER WAR GAME

- SEALIFT -

by

Walter J. Kirsch III

This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

IN

OPERATIONS RESEARCH

from the

United States Naval Postgraduate School

ABSTRACT

The Center for Naval Analyses Computer War Game, SEALIFT, is a Monte-Carlo simulation designed to help study sealift capabilities in an ASW environment. A mathematical model of the SEALIFT game is posited to obtain expected value results approximating those of the SEALIFT game. The model is cast in the Fortran terminology of SEALIFT and an effort is made to accurately reflect the SEALIFT flow chart logic in its development. Comparisons with SEALIFT results to determine the model's reliability and accuracy are not made.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1. Introduction

Objective. In 1963-1964 the Center for Naval Analyses, CNA, of the Franklin Institute constructed a rather large and complex computer war game, SEALIFT I [1]. This game was used as an analytical tool for the Anti Submarine Warfare Study, CYCLOPS II. Due to the large computer running time of SEALIFT an after the fact need for a "quick and dirty" approximation to actual SEALIFT results was felt, and during the summer of 1964 this author was privileged to work with the CNA CYCLOPS study group on that approximation. This thesis is an outgrowth of that summer's work. The objective of this thesis is to provide the logic for an expected value analysis of that portion of the SEALIFT game directed toward calculating transport ship losses through submarine action. This thesis is then a "first cut" model to approximate a model, and the analysis is executed under simplifying assumptions.

Background. Functionally, the SEALIFT game pits the submarine forces of one opponent against the sea transport endeavors of another opponent in support of a time and scope limited war overseas. The major game output is the number of transport ships sunk during the war and, through inference, the effect of these losses on the sealift supply rates.

Structurally, SEALIFT is an event store Monte-Carlo computer simulation and is primarily probabilistic with essentially no kinematics or geometry wired in. Although certain tactical procedures were assumed during construction, they may be circumvented or modified by the input parameter set. This set is on the order of four hundred and contains numerous decision switches as well as numerical data. It is in the scope of this parameter set that one finds the flexibility of the game and, unhappily, the bane of forthright analysis.

The focal point of SEALIFT is the submarine-convoy encounter and of the thirty-two subroutines which comprise SEALIFT, five form the substance of this event. The remaining routines are directed toward making the game more realistic by introducing other strategies and toward controlling the flow of the game. Figure 1 is a graphical representation of the game. A general and more detailed description of the game's structure and logic is quoted from SEALIFT I [1] in Appendix I.

Philosophy. All submarine loss events in SEALIFT except those incurred in the submarine-convoy encounter are analyzed here utilizing independent Bernoulli trial techniques to yield simple expected value results. To effect this result, several event-time dependent routines were omitted from consideration and a few were modified to simplify the analysis.

Preliminary investigation indicated that the submarine-convoy interaction was the most sensitive in the determination of transport ship losses and that simple techniques were not sufficiently accurate to determine these losses. This encounter has been recast in the framework of a Markov chain where each of the sequences of events which may befall a submarine upon convoy passage is considered a state of the Markov chain. Primary effort has been directed toward accurately effecting the Markov structure, and minimal effort toward the accurate reflection of each state within the chain. Each state, i.e., sequence of events, is described via independent Bernoulli trial terminology, again with approximations.

This analysis has been made with some trepidation. It may be argued that detailed Markov analysis is pretentious in view of the coarse Bernoulli approximations; nevertheless, if the results of this expected value analysis approximate SEALIFT results, it is hoped this method of analysis will provide new insight for a more detailed expected value analysis of the entire SEALIFT game.

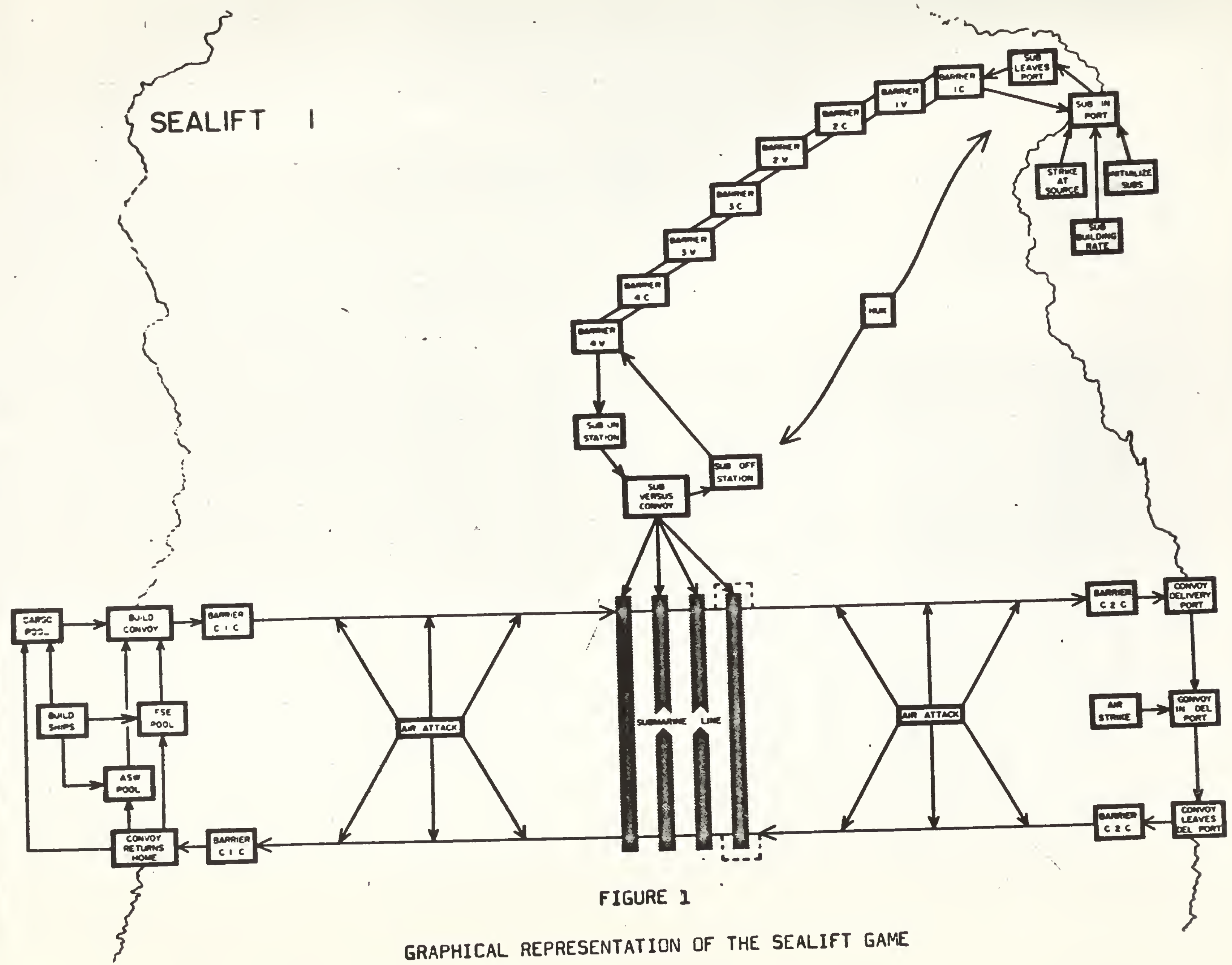

FIGURE 1

GRAPHICAL REPRESENTATION OF THE SEALIFT GAME

2. Mathematical Preliminaries

Notational usage in this thesis. The following symbols are used in this thesis:

Symbol	Definition
$Pr(A)$	The probability of the event A.
$Pr(A/B)$	The probability of the event A conditioned on the occurrence of the event B.
$E(N)$	The expected value of the random variable N.
$\left[a\right]$	The greatest integer less than or equal to a. The brackets are used, additionally, to enclose a matrix when its elements are listed in full.
(q_{ij})	The matrix whose ij th element is q_{ij}.
$A \overset{d}{=} B$	A is defined as B, or A is identically equal to B.
$A \overset{a}{=} B$	A is approximately equal to B.
$\prod_{i=1}^{n} A_i$	The product, $A_1 \times A_2 \times \ldots \times A_n$.
$\sum_{i=1}^{n} A_i$	The sum, $A_1 + A_2 + \ldots + A_n$.
iff	if and only if

Markov chains. It is generally understood that if a physical system can exist in a known set of stable states, say E_1, E_2, ..., (finite or countable), and if with each state there is associated a probability, $Pr(E_i) = p_i$, that the system is in that state (or goes to that state), then the probability that the system transits the states E_{j_0}, E_{j_1}, ..., E_{j_n} is the product $\prod_{k=0}^{n} p_{j_k}$. The basic assumption here is transitional independence, i.e., the chance for transition from a given state to another does not depend upon the given state or upon past states.

Some physical systems cannot be so explained but require the dependence of transitional probabilities upon the history of the system. The simplest of such dependencies is that the next state in sequence depends upon the present or given state only. In this case the outcome, E_k, is no longer associated with a fixed probability p_k, but every pair (E_i, E_j) is associated with a conditional probability, p_{ij}, which denotes the chance that the system goes to state E_j given that it was in state E_i at the previous step. In general, the transitional probability p_{ij} is a function of the sub-subscript in the transit E_{j_0}, ..., E_{j_n}, and the process is called homogeneous if p_{ij} is independent of this sub-subscript. (Only the homogeneous case will be considered in the expected value analysis and in what follows.)

If, in addition, the probabilities, p_i, of the initial trial i are known, the probability that the system transits the states E_{j_0}, E_{j_1}, ..., E_{j_n} is the product $p_{j_0} \times \prod_{k=0}^{n-1} p_{j_k j_{k+1}}$. The conditional probability of the transit, given the initial state E_{j_0}, is then $\prod_{k=0}^{n-1} p_{j_k j_{k+1}}$.

The transitional probabilities can be arranged in a matrix, called the transition probability matrix, P

$$(2.0) \qquad P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots \\ p_{21} & p_{22} & p_{23} & \cdots \\ p_{31} & p_{32} & p_{33} & \cdots \\ \vdots & \vdots & \vdots & \cdots \\ \vdots & \vdots & \vdots & \cdots \\ \vdots & \vdots & \vdots & \cdots \end{bmatrix}$$

where the standard matrix subscripts ij also stand for transition from state E_i to state E_j. Together with the initial distribution, p_i, the matrix completely defines a Markov chain with states E_1, E_2, ... It should be noted that the elements of each row must sum to one, and that the element p_{ij} denotes the chance for transition from state E_i to state E_j in one step.

The natural extension is to investigate the concept of transition from state E_i to state E_j in more than one step, say in n steps. This transition can occur via many paths, and hence the conditional probability, $p_{ij}^{(n)}$, of finding the system in state E_j at the n+r th step, given that it was in state E_i at the r th step, is the sum of the conditional probabilities for each possible path. In particular,

$$p_{ij}^{(1)} = p_{ij}$$

$$p_{ij}^{(2)} = \sum_r p_{ir}p_{rj}$$

The equation [1]

$$(2.1) \qquad p_{ij}^{(n)} = \sum_r p_{ir}p_{rj}^{(n-1)}$$

and the Chapman-Kolmogorov equation [1]

$$(2.2) \qquad p_{ij}^{(m+n)} = \sum_r p_{ir}^{(m)} p_{rj}^{(n)}$$

follow by mathematical induction.

---

[1] William Feller, An Introduction To Probability Theory And Its Applications (New York: John Wiley and Sons, 1957) p. 348.

The probabilities, $p_{rj}^{(n-1)}$, can be arranged in a matrix of identical structure as that given by equation (2.0). Coupled with equation (2.1), this suggests matrix multiplication - the ij th element of the n step matrix is the inner product of the i th row of the one-step matrix with the j th column of the (n-1) st step matrix, i.e.,

$$P^n = P \times P^{n-1}$$

Similarly, equation (2.2) implies

$$P^{m+n} = P^m \times P^n$$

Also, if p_i is the probability of the initial state E_i, the unconditional probability of finding the system in state E_j after n steps is

(2.3) $$p_j^{(n)} = \sum_i p_i \times p_{ij}^{(n)}$$

<u>A particular n step transition matrix</u>. The expected value analysis requires the n step matrix, P^n, formed from the (one-step) transition probability matrix, P,

(2.4) $$P = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 \\ p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = (p_{ij})$$

where $p_1 = p_{11} = p_{21}$ of the previous notation, etc. If A_1, A_2, A_3, A_4 are defined such that

$$A_1 = \begin{bmatrix} p_1 & p_2 \\ p_1 & p_2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} p_3 & p_4 \\ p_3 & p_4 \end{bmatrix}, \quad A_3 = \begin{bmatrix} p_5 & p_6 \\ p_5 & p_6 \end{bmatrix}, \quad A_4 = \begin{bmatrix} p_7 & p_8 \\ p_7 & p_8 \end{bmatrix},$$

then

$$P = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 \\ \emptyset & I & \emptyset & \emptyset \\ \emptyset & \emptyset & I & \emptyset \\ \emptyset & \emptyset & \emptyset & I \end{bmatrix}$$

and

$$(2.5) \qquad P^2 = \begin{bmatrix} A_1^2 & (A_1+I)A_2 & (A_1+I)A_3 & (A_1+I)A_4 \\ \emptyset & I & \emptyset & \emptyset \\ \emptyset & \emptyset & I & \emptyset \\ \emptyset & \emptyset & \emptyset & I \end{bmatrix}$$

and in general

$$P^n = \begin{bmatrix} A_1^n & (\sum_{i=0}^{n-1} A_1^i)A_2 & (\sum_{i=0}^{n-1} A_1^i)A_3 & (\sum_{i=0}^{n-1} A_1^i)\, A_4 \\ \emptyset & I & \emptyset & \emptyset \\ \emptyset & \emptyset & I & \emptyset \\ \emptyset & \emptyset & \emptyset & I \end{bmatrix}$$

By recursive multiplication, it follows that

$$A_1^n = (p_1 + p_2)^{n-1} \times A_1, \quad n \geqq 1$$

and therefore

$$\sum_{i=0}^{n-1} A_1^i = \sum_{i=1}^{n-1} A_1^i + I = \sum_{i=1}^{n-1} (p_1 + p_2)^{i-1} \times A_1 + I$$

$$= \frac{1 - (p_1 + p_2)^{n-1}}{1 - (p_1 + p_2)} \times A_1 + I$$

Letting

$$f_2(n) = \frac{1 - (p_1 + p_2)^{n-1}}{1 - (p_1 + p_2)} \; ; \quad f_1(n) = \frac{1 - (p_1 + p_2)^n}{1 - (p_1 + p_2)} ,$$

it follows that

$$\sum_{i=0}^{n-1} A_1^i = \begin{bmatrix} p_1 \, f_2(n) + 1 & p_2 \, f_2(n) \\ p_1 \, f_2(n) & p_2 \, f_2(n) + 1 \end{bmatrix}$$

and substitution in (2.4) obtains

$$(2.6) \qquad P^n = \begin{bmatrix} (p_1 + p_2)^{n-1} \times \begin{bmatrix} p_1 & p_2 \\ p_1 & p_2 \end{bmatrix} & f_1(n) \times \begin{bmatrix} p_3 & p_4 & p_5 & p_6 & p_7 & p_8 \\ p_3 & p_4 & p_5 & p_6 & p_7 & p_8 \end{bmatrix} \\ \emptyset_{6\times 2} & I_{6\times 6} \end{bmatrix}$$

3. The Model for the Expected Value Analysis

In what follows, a perusal of Appendix I and of SEALIFT I [1] is assumed.

Methodology. The heart of any computer game is the flow chart logic, and it was via these charts that SEALIFT was submitted to analysis. Prior to detailed analysis, the game was examined as a unit to cull those routines pertinent to the submarine from those of a bookkeeping nature and other routines of non-interest, including those which determined convoy losses to non-submarine causes. Of the thirty-two routines which comprise the SEALIFT game, twenty were discarded as irrelevant to the objective of this thesis. The remaining routines were again scrutinized for syntax as related to a typical play of the game. From the observed functional and logical arrangement of these routines, the all important abstraction from "real world," SEALIFT, to model was attempted.

Concurrent with, but not independent of, this abstraction, a variable which would be definitive to the expected value analysis was sought. Bearing in mind that the motive was to evaluate losses to the cargo fleet by submarine offensive, the natural and obvious choice was "the total expected number of torpedoes fired at cargo (vice convoy) ships during the war." As stated, this number proved to be unattainable, the primary culprit being the time and event dependencies which permeate (are) the game. It appeared, however, that if the number of torpedoes a typical submarine would fire at cargo ships during a single cycle at sea, i.e., from home port to station to home port, could be determined, this number could be extended into a total war figure. The intuitive appeal of this single cycle approach is that, save for the submarines initially on station, it removed, postponed really, the necessity for considering the element of time. It rendered

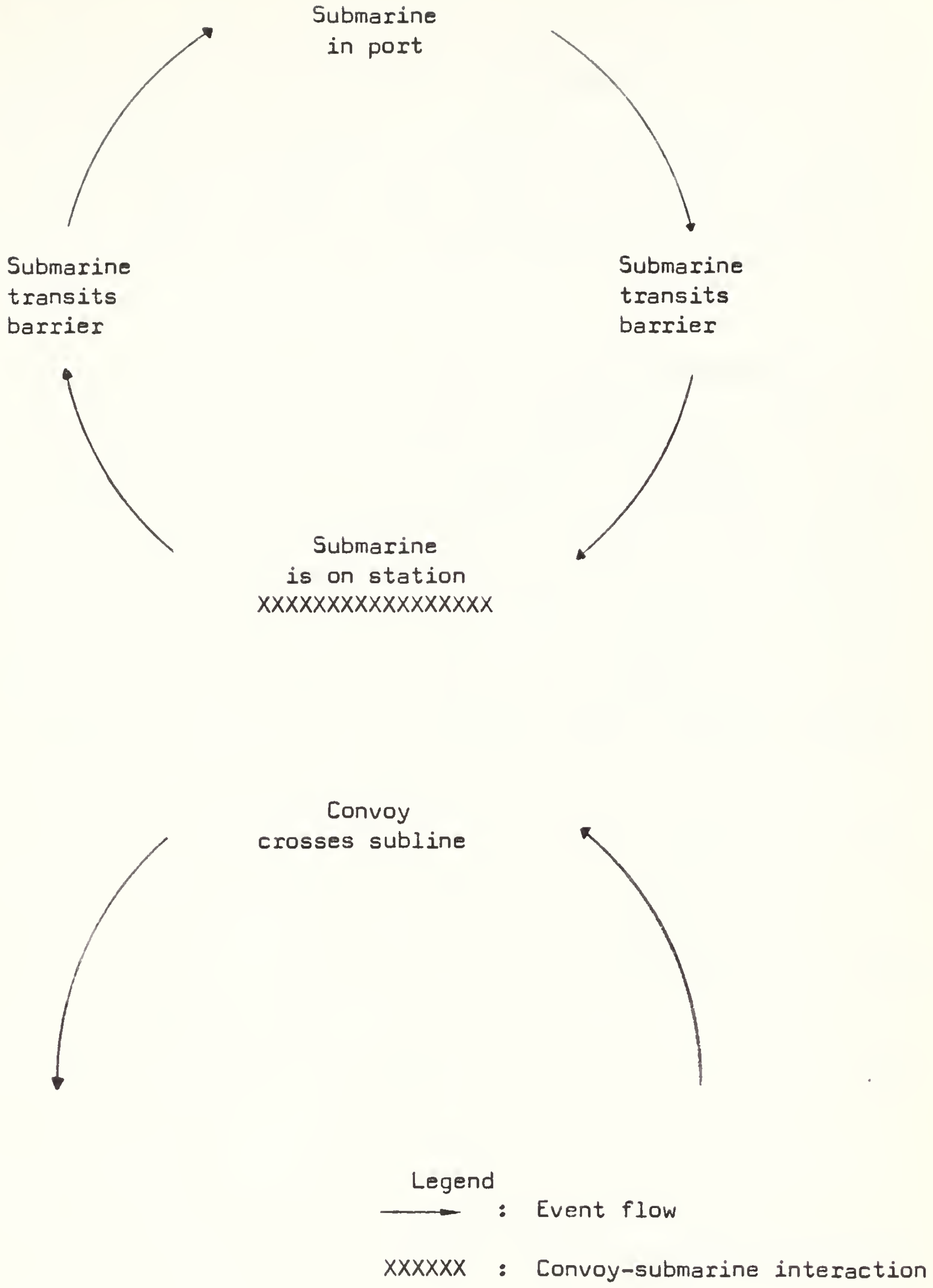

FIGURE 2

GRAPHICAL REPRESENTATION OF THE CYCLE ABSTRACTION

FROM SEALIFT TO THE EXPECTED VALUE MODEL

individual submarines' unique and time dependent lives indistinguishable and it collapsed the immense and complicated time-event structure into a fairly tractable entity. The basic cycle is represented in Figure 2. The model for the expected value analysis developed in this thesis is therefore the logic needed, using the Markov chain process, to arrive at the number of torpedoes a submarine would fire at cargo ships during a single cycle at sea.

In developing this logic, three difficulties became immediately apparent:

1. Aside from the three different type submarines and the four possible sonar boxes to which they could be assigned, the submarines initially on station are unique. These submarines have a probability distribution on their departure-from-on-station time, unlike other submarines which have a fixed mission time. This probability distribution necessitated a modification of the associated Markov chain, since prior SEALIFT results indicated this cycle to be a very sensitive factor.

2. How are the results of a single cycle at sea "extended" to cover the total war? In particular, what assumptions are required to minimize the effects of a coarse approximation? How are excessively small, or large, convoy losses to be interpreted?

3. How can the event and time dependencies within a single cycle be handled effectively?

The third difficulty was resolved by essentially ignoring the event and time dependencies inherent to SEALIFT. For the most part, these dependencies proved incompatible with either the cycle abstraction or with the Markov chain, or both. Where possible, however, they were included within the framework provided by this thesis.

The second difficulty was resolved by determining the minimum number of

cycles a live submarine would make during the war. The basic cycle was then repeated that minimum number of times with surviving submarines from one cycle serving as inputs to the following cycle. Rationale for this choice was provided by two facts. First, a priori information from the SEALIFT game indicated that where the submarine was almost immediately overwhelmed, the significant action took place early in the war. In this case, the number of cycles would prove immaterial. If, however, the convoys were continually decimated, this expected value analysis would show abnormally high convoy losses, and this would actually serve the purpose intended by SEALIFT.

If opposing forces were fairly balanced, successive cycles would show nearly identical convoy losses, again providing an indication of the progress of the war. In other words, under the assumptions that the omitted time and event dependent events are not significant, the analysis provided herein should reflect the trends of a corresponding SEALIFT game. This method of extension was also supported by the fact that, almost without exception, the omission of each of the dependent events effects an over-estimation of the number of cargo ship losses. The choice of a minimum number of cycles would then partially compensate for this over-estimation.

The theoretical resolution of the first difficulty proved trivial, falling under the cognizance of the integral form of the theorem of total probability. The evaluation of this integral proved to be computationally messy. The functional simplicity of the probability distribution and a linear approximation to the step function integrand allowed a simple but accurate approximation to this integral. Each of these difficulties is discussed more fully in subsequent subsections.

Having chosen a tentative model to represent the game, this model was

divided into its component parts and separately analyzed. The basic cycle could be broken into two phases, (1) the submarine-barrier penetration, and (2) the submarine-convoy interaction, the Markov chain of this development. The probabilities of the sequences of events associated with each phase, i.e., the barrier penetration and each state of the Markov chain, were analyzed via the corresponding SEALIFT subroutines or parts of subroutines. The SEALIFT subroutine logic was reformulated in the structure of a tree or truth table. While an attempt was made to construct the trees to match the subroutine logic exactly, some simplication was employed.

Basic assumptions used in the expected value analysis. Perhaps the most definitive assumption made in the expected value analysis is that the submarine-convoy encounter should be the significant event for determining cargo ship losses. By implication, then, the convoy losses to air attack, both at sea and in port, and by barrier penetration are assumed small enough so as not to render the submarine target-limited, vice torpedo-limited, and also that each convoy be initially large enough to comply with this restriction. Thus, Event 3 (Convoy enters barrier C1C), Event 4 (Air attack against convoy), Event 6 (Convoy enters barrier C2C), Event 7 (Convoy in delivery port), Event 8 (Convoy enters barrier C2C on return trip), and Event 9 (Convoy enters barrier C1C on return trip) were classified as irrelevant to the analysis for this reason.

This analysis hinges, then, on the number of submarines which penetrate the convoys, and prior to that, the number of submarines which reach the subline. For this reason, all the submarine loss events were deemed important. In the subsequent analysis, however, it was observed that the train of events which could occur to one submarine was dependent upon previous events involving other submarines. In other words, the stochastic process

associated with a particular submarine was dependent upon the realization of the process associated with other submarines on station with it and submarines which proceeded it to station. Such dependencies are inherently intractable to analysis and they were ignored in the expected value model. By their omission, these dependencies introduced a cumulative over-estimation of the number of cargo ship losses. In particular, Event 17 (Submarine enters variable barrier) calls the subroutine VARDET to recalculate the reduced barrier-detects-submarine probability in the event a submarine sinks one or more of the barrier ships. Following submarines would then stand a better chance of slipping through that barrier until the loss was replaced (Event 22 (Replace a barrier ship)). A concurrent effect is the torpedo usage on barrier ships. This expenditure directly affects the ammunition available to be used on cargo ships later in the submarine's cycle. This last effect could become more invidious during a HUK attack, Event 15 (Submarine versus HUK group). In this routine the only ways a submarine could escape, if detected by the HUK Group, are if her time or ammunition runs out or if her torpedoes miss their mark! In addition to introducing event dependencies, the HUK attack event presented further theoretical difficulties in the form of time dependencies. Each submarine, while at sea, is subject to HUK attack every HUKHR interval. This event meshes with every other event in the submarine's life and tends to hopelessly confuse expected value analysis.

The subroutines FSEDET and ASWDET, which perform the same function for the forward screen escort and the ASW screen as VARDET does for the barrier ships, are called during the play of Event 19 (Submarine versus convoy). Like VARDET, these routines influence the train of events of following submarines. Again, parasitic torpedo usage is encountered; however, an attempt was made to approximate this usage in the model in the case of the forward

screen escort.

Another, and time related, dependency was the possible monthly change in submarine-detects-convoy probability for the submarines stationed in the fourth sonar box, a portion of Event 5 (Convoy arrives at submarine line). This implied an incompatibility with the cycle concept. Only fortuitous combinations of the number of cycles of the war, duration of the war and constancy of the detection probability associated with the fourth sonar box during each cycle would permit easy inclusion of this dependency.

Similarly, Event 13 (Strike at submarine source) was a time dependent event. Provision for up to ten air strikes, at times chosen through the input parameter set, were made in SEALIFT. These air strikes, too, are incompatible with the cycle concept used in the model.

To summarize, the routines ASWDET, FSDET, VARDET, Event 15 (Submarine versus HUK group), Event 22 (Replace a barrier ship), Event 13 (Strike at submarine source), and a portion of Event 5 (Convoy arrives at the submarine line) were ignored by virtue of their intractability to analysis.

Finally, Event 12 (Build submarines) provides for a monthly submarine building rate. While this event is also time dependent, it was adapted to the model by applying the number of submarines built during the entire war as an additional input to the first cycle, a striking illustration of the time independence of the cycle concept.

It should be noted that, in SEALIFT, there exist at least two levels of logic, the internal event logic which computes losses to convoys, submarine losses, etc., and the logic which controls the flow of the game, i.e., computes when and how events will occur. These levels are frequently meshed within each subprogram, and it is with regard to the first named level that routines were ignored or classified as irrelevent. The second type logic

was used, implicitly, whenever necessary, in the analysis. In addition, the following were ignored as being primarily bookkeeping type routines: Event 1 (Build ships), Event 2 (Form convoy), Event 10 (Convoy dispersed at home), Event 14 (Submarine leaves port), Event 18 (Submarine arrives on station), Event 20 (Submarine goes home), Event 21 (Submarine arrives home), and Event 24 (Replace convoy ships).

The events from SEALIFT that form the basis for the analysis are therefore: Event 5 (Convoy arrives at submarine line), Event 11 (Initialize submarines), Event 16 (Submarine enters constant barrier), Event 17 (Submarine enters variable barrier), and Event 19 (Submarine versus convoy).

3.1 Markov Chain Structure for the Expected Value Analysis

For a given submarine, the initial Markov chain begins after that submarine's first convoy has passed the subline. The initial states for this Markov chain are the possible sequences of events which could have occurred to the submarine when that convoy passed. These states may be represented by the arborescence, with root "Submarine on Station," exhibited in Figure 3 where a state of the Markov chain is defined as the path from the root to a terminal point. Letting E_i denote the states of interest, define

E_1 = submarine does not detect the convoy.

E_2 = submarine detects the convoy, does not penetrate the Forward Screen Escort (FSE), and is not killed.

E_3 = submarine detects the convoy, penetrates the FSE, penetrates the inner screen (ASW), shoots one salvo at cargo ships, and survives any resulting Search and Attack Unit (SAU) action.

E_4 = submarine detects the convoy, penetrates the FSE, penetrates the ASW, shoots one salvo at cargo ships, and is killed in resulting SAU action.

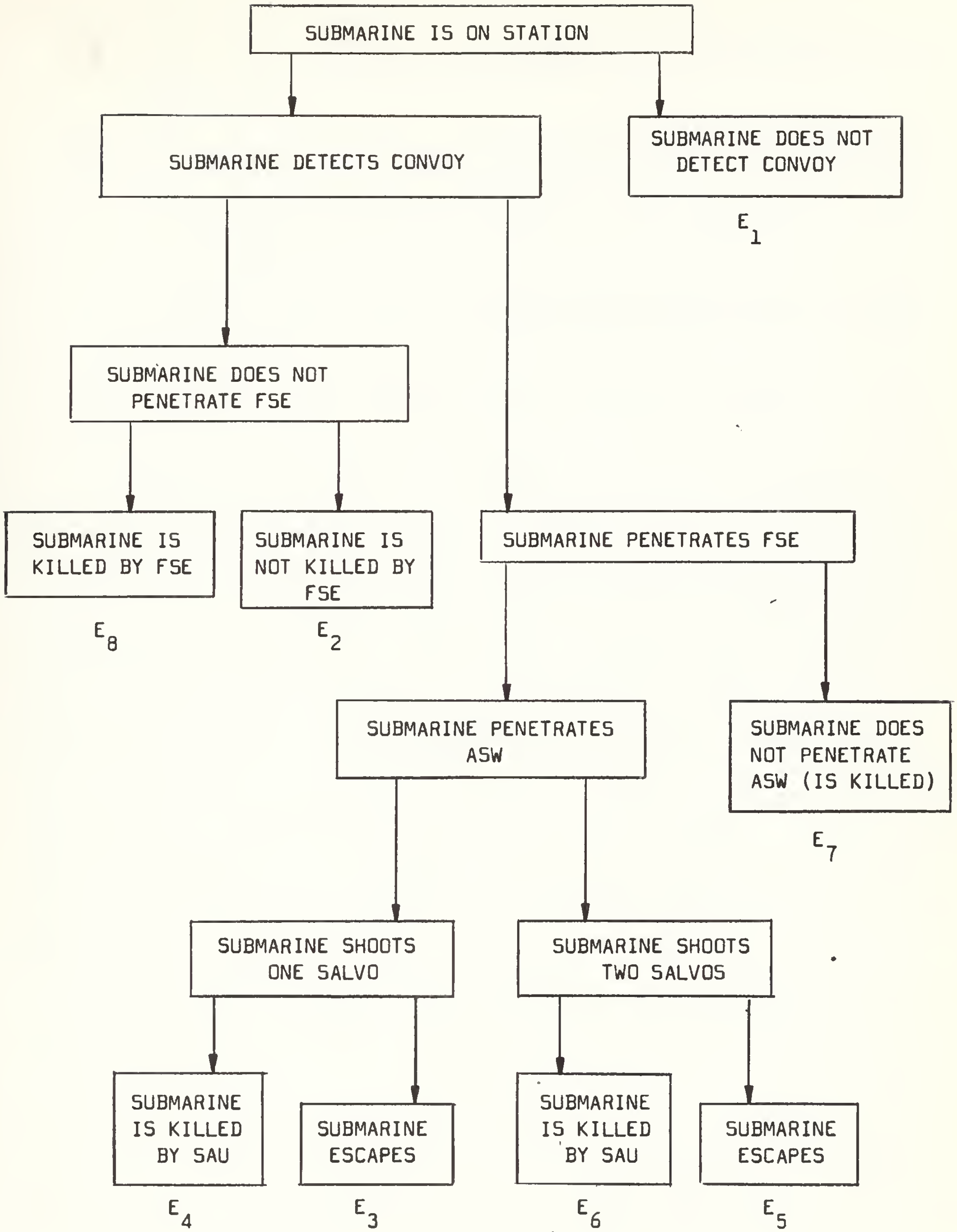

FIGURE 3

ARBORESCENCE ASSOCIATED WITH THE STATES OF THE FIRST MARKOV CHAIN

E_5 = submarine detects the convoy, penetrates the FSE, penetrates the ASW, shoots two salvos at cargo ships, and survives any resulting SAU action.

E_6 = submarine detects the convoy, penetrates the FSE, penetrates the ASW, shoots two salvos at cargo ships, and is killed in the resulting SAU action.

E_7 = submarine detects the convoy, penetrates the FSE, and is killed by the ASW.

E_8 = submarine detects the convoy and is killed by the FSE.

This Markov chain allows for the possibility that if a single salvo does not exhaust the torpedo supply of a submarine during an attack, there is a chance that the submarine will be unable to fire again at that convoy and will, therefore, return to station with a partial load of torpedoes to await the arrival of the convoys. It is assumed in this model, however, that two salvos will exhaust a submarine's torpedo supply. This assumption is equivalent to the assumption that the necessary train of events which would place a submarine in the position of having fired more than two single salvos at as many different convoys to be very improbable, and neglects this possibility.

The occurrence of states E_1 and E_2 imply that the submarine returns to station where she awaits the next convoy arrival, which is to say, she remains alive in this Markov chain. State E_5 is followed by the submarine's attempt to return home, and is, therefore, an absorbing state. Similarly, states E_4, E_6, E_7, and E_8 are absorbing since the submarine is killed. State E_3 is absorbing for another reason; it is the entry point for the second Markov chain described below.

If the probability of the initial states are denoted by $Pr(E_i) = p_i$,

the one step transition probability matrix is given by equation (2.4). If a total of n+1 convoys pass the subline during a submarine's allotted on-station time, then the transition matrix, P, must be raised to the n th power since the initial states are caused by passage of the first convoy to yield the n step transition matrix P^n. The resulting matrix, P^n, is given by equation (2.6). This matrix and the initial probability distribution, p_i, suffice to determine the unconditional probabilities, $P_j^{(n)}$, that a submarine will be in each of the possible states at the completion of n steps, (n+1) convoys having passed. The unconditional probabilities, $P_j^{(n)}$, are obtained from equation (2.3) and from them, the probabilities of certain relevant events can be obtained:

(3.0) $P(A_1) \overset{d}{=}$ Pr (a submarine survives this Markov chain to attempt passage home/ n+1 convoys have passed the subline)

$$= P_1^{(n)} + P_2^{(n)} + P_5^{(n)}$$

(3.1) $P(A_2) \overset{d}{=}$ Pr (a submarine fires one salvo at the cargo ships in this Markov chain/ n+1 convoys have passed the subline)

$$= P_3^{(n)} + P_4^{(n)}$$

(3.2) $P(A_3) \overset{d}{=}$ Pr (a submarine fires two salvos at the cargo ships in this Markov chain/ n+1 convoys have passed the subline)

$$= P_5^{(n)} + P_6^{(n)}$$

(3.3) $P(B_r) \overset{d}{=}$ Pr (a submarine is absorbed in state E_3 upon passage of the r th convoy)

$$= p_3 (p_1 + p_2)^{r-1}, \quad r = 1, \ldots, n + 1$$

The second Markov chain is almost identical to the first chain and is concerned with those submarines which are absorbed in state E_3. These submarines return to station having fired a single salvo of torpedoes at the previous convoy. Assuming that a total of (n+1) convoy cross the sub-line, the number of steps of the second Markov chain depends upon which convoy caused a submarine to be absorbed in state E_3. If a submarine is absorbed in state E_3 at the passage of the r th convoy, the (r+1)st convoy "initializes" the second Markov chain and there are $(n+1) - (r+2) + 1 = n-r$ steps in the second chain. The probability that a submarine enters the second Markov chain "just after" passage of the r th convoy is given by equation (3.3).

The second Markov chain is described by its states:

F_1 = submarine does not detect the convoy.

F_2 = submarine detects the convoy, does not penetrate the FSE, and is not killed.

F_3 = submarine detects the convoy, penetrates the FSE, penetrates the ASW, shoots her remaining torpedoes at cargo ships, and survives any resulting SAU action.

F_4 = submarine detects the convoy, penetrates the FSE, penetrates the ASW, shoots her remaining torpedoes at cargo ships, and is killed in the resulting SAU action.

F_5 = submarine detects the convoy, penetrates the FSE, and is killed by the ASW.

F_6 = submarine detects the convoy, and is killed by the FSE.

Denoting the probabilities of the initial states of this Markov chain by $Pr(F_i) = q_i$, the one step transition probability matrix is then

$$Q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 \\ q_1 & q_2 & q_3 & q_4 & q_5 & q_6 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = (q_{ij})$$

and the n-r step transition matrix, by analogy with equation (2.6), is

$$Q^{n-r} = \begin{bmatrix} (q_1 + q_2)^{n-r-1} \times \begin{bmatrix} q_1 & q_2 \\ q_1 & q_2 \end{bmatrix}, & \dfrac{1-(q_1+q_2)^{n-r}}{1-(q_1+q_2)} \times \begin{bmatrix} q_3 & q_4 & q_5 & q_6 \\ q_3 & q_4 & q_5 & q_6 \end{bmatrix} \\ \emptyset_{4\times 2} & I_{4\times 4} \end{bmatrix} = (q_{ij}^{(n-r)})$$

If $R_j^{(n)}$ is defined as the unconditional probability that the submarine is in state F_j after n steps, then, using the theorem of total probability,

$$R_j^{(n)} = \sum_{r=1}^{n+1} \Pr(\text{a submarine is in state } F_j \text{ after } n+1 \text{ convoys/she entered the second chain after the } r \text{ th convoy has passed}) \times \Pr(\text{she entered the second chain after the } r \text{ th convoy}).$$

$$= \sum_{r=1}^{n} Q_j^{(n-r)} \times P(B_r), \quad n \geqq 1$$

$$= \sum_{r=1}^{n} \sum_{i=1}^{6} q_i\, q_{ij}^{(n-r)}\, p_3\, (p_1 + p_2)^{r-1}, \quad n \geqq 1$$

where

$$q_{ij}^{(o)} \overset{d}{=} \delta_{ij}, \text{ the Kronecker delta.}$$

The probabilities of the remaining events of relevance can now be obtained as:

(3.4) $P(A_4) \overset{d}{=}$ Pr (a submarine survives the submarine-convoy interaction/ n+1 convoys have passed the subline)

$$= P_1^{(n)} + P_2^{(n)} + P_5^{(n)} + R_1^{(n)} + R_2^{(n)} + R_3^{(n)} + p_3(p_1 + p_2)^n$$

and

(3.5) $P(A_5) \overset{d}{=}$ Pr (a submarine fires her remaining salvo in the second Markov chain/ n+1 convoys have passed the subline)

$$= R_3^{(n)} + R_4^{(n)}$$

In SEALIFT, the submarines may be separated into two sets, those which are on station at the beginning of the war and have a departure time distributed uniformly on an interval, and those submarines which arrive on station after the war starts and have a fixed mission time. Once the game parameters have been set the arrival of each convoy is deterministic. As a consequence, the variable n+1 is a step function of the submarines departure time T, i.e., n+1 = g(T). Representing the equations, (3.0), (3.1), (3.2), (3.4) and (3.5) symbolically as

$Pr(A/n+1)$ = Pr (of the event A given n+1 convoys have passed)

this probability becomes $Pr(A/g(T))$. Invoking the theorem of total probability, Pr(A) can be found as

$$(3.6) \qquad Pr(A) = \int_t Pr(A/g(t))\, d\, Pr(T \leqq t)$$

This equation is valid for both sets of submarines although trivial for those with a fixed mission time. For the submarine with a fixed mission time Pr(A) is simply the integrand of this equation evaluated at the upper limit

of t. The evaluation of equation (3.6) is crucial in determining the expected number of cargo ships sunk per cycle.

This completes the construction of the Markov chain for a generalized submarine. To apply this structure to the SEALIFT game, the integrals Pr(A) and the transition probabilities p_{ij} and q_{ij} must be expressed in terms of of the FORTRAN variables of SEALIFT. The submarine barrier penetration must be analyzed in like terms. Finally, the evaluated terms must be synthesized to answer the basic question "What is the expected number of torpedoes fired at cargo ships per cycle?" and its extension "What is the expected number of torpedoes fired at cargo ships in the war?"

3.2 Detailed Analysis in Terms of the FORTRAN Variables of the SEALIFT Game.

The detailed analysis to follow will be executed in the following order, (1) evaluation of the integrals Pr(A), (2) evaluation of the barrier penetration, the transition probabilities, and related expected number, (3) synthesis to determine the expected number of torpedoes fired at cargo ships during the war. The following list of SEALIFT I [1] FORTRAN variables is pertinent to the analysis, and the following convention is observed: The letter "I" is reserved for the type submarine index; however, it is employed in two ways. As a subscript, e.g., ALTIM (I), "I" can take on the values 1, 2, and 3. As a part of a variable name, e.g., NSUB'I'T, "I" is set off in this analysis by single quotation marks and can take on the values 1, 2 and N. In both usages, the values of "I" denote conventional number 1, conventional number 2, and nuclear submarines, respectively.

Variable Name	Definition
TBCON	Time between each convoy sailing (hours).
CONSPD	Speed of convoy (knots).

CONHR	Time after start of war that the first convoy leaves home port in hours.
ONEWAY	One way convoy distance from home port to delivery port (n. miles).
ULTIME	Time convoy spends in delivery port.
ALTIM(I)	Allowable time in hours for type I submarine before it must leave its station.
DSUBTA	Distance from submarine home port to submarine station (n. miles).
SUBSPD(I)	Speed of type I submarine.
NSUB'I'T	Number of type I submarines at start of war.
FSUB'I'OS	Fraction of type I submarine on station at start of war.
P'I'BOX(J)	Probability that a type I submarine on station will be in BOX # J, J = 1, ..., 4.
NSUB'I'	Number of type I submarines built per month.
PBARSC(I,J)	Probability that a type I submarine is sunk by the Jth constant barrier, J = 1, ..., 4.
PBARSV(I,J)	Probability that a type I submarine is sunk by the Jth variable barrier, given it was detected by this barrier, J = 1, ..., 4.
PVBDET(I,J)	Probability that a type I submarine is detected by the Jth variable barrier, with barrier at its full quota of ships.
VAIRDET(J)	Part of detection probability of variable barrier J that is not supplied by ships within that barrier which might be sunk by a submarine. Thus, if a mixed barrier of aircraft and ships were used and all the ships were sunk, the probability of detection drived from the air barrier (VAIRDET(J)) would still be present. The "total" detection probability of a variable barrier is PVBDET(I,J) + VAIRDET(J). For a ship barrier only, VAIRDET(J) should be set to zero.
RGDT(K,I)	Maximum athwartship detection range of type I submarine against convoy type K sonar conditions K=1 iff good sonar conditions K=2 iff poor sonar conditions
SUBLINE	Length of submarine line (n. miles), i.e., width of patrol area.
TWAR	Duration of war (hours).

FAIRSC	Is forward escort screen made up entirely of aircraft (non-sinkable)? (YES=1, NO=2).
ATSFE(I)	Number of times a type I submarine can try to repenetrate forward screen.
PDFS(I,K)	Probability that a type I submarine in type K sonar conditions (denoted by "(I,K) submarine" henceforward) is detected by forward screen escort while penetrating screen with screen at its full quota of ships.
PDFER	Part of detection probability of forward screen that is not supplied by ships within that screen which might be sunk by a submarine. Thus, if a mixed screen of aircraft and ships were used and all the ships were sunk, the probability of detection derived from the air screen (PDFER) would still be present. The "total" detection probability of the forward screen is then PDFS(I,K) + PDFER. For a ship's screen only, PDFER must be set to zero.
PDFCE(I,K)	Probability that an (I,K) submarine is sunk by forward screen escort, given submarine has been detected by same.
PSFSE(I)	Probability that a type I submarine shoots at forward screen escort ship in an attempt to break contact.
PFCES(I,K)	Probability that an (I,K) submarine sinks forward screen escort ship, given it decides to shoot at it.
PSFCE(I,K)	Probability that an (I,K) submarine is sunk by forward screen escort ship, given this submarine has already sunk a forward screen escort ship, i.e., flaming datum.
ASALVO(I)	Number of anti-ASW torpedoes used per salvo by a type I submarine.
PDETU(I,K)	Probability that an (I,K) submarine is detected by the ASW inner screen with direct path sonar, given it was not detected previously with bottom bounce sonar.
VA(K)	Conversion factor reflecting improved ASW inner screen detection probability with direct path sonar, in type K sonar conditions, because of previous bottom bounce detection.
PBB(I,K)	Probability that an (I,K) submarine is detected by the ASW inner screen with bottom bounce sonar.
PKBB(I,K)	Probability that an (I,K) submarine is killed, given that it is attacked or re-attacked following a detection with bottom bounce sonar.

PAT(I,K)	Probability that an (I,K) submarine is attacked given that it has been detected with bottom bounce sonar only.
PCC(I,i)	Probability that a group from the ASW inner screen re-attacks a type I submarine at the i th opportunity in action resulting from a bottom bounce detection, i=1, 2, 3.
PABB(I,K)	Probability that an (I,K) submarine attacks a group from the ASW inner screen in action resulting from a bottom bounce detection.
PKBBG(I)	Probability that a type I submarine sinks a ship in the group from the ASW inner screen in action resulting from a bottom bounce detection.
QTC 177	Designator for possibility that an engagement between the ASW inner screen and a submarine is terminated after an unsuccessful submarine counter attack, e.g., due to quiet torpedoes used by submarine (QTC equal to zero implies engagement is to continue; QTC equal to one implies engagement is terminated. No other values are possible).
PKINS(I)	Probability that a type I submarine is sunk by the ASW inner screen during action following a detection with direct path sonar.
PSCA(I,i)	Probability that an ASW inner screen re-attacks a type I submarine at the i th opportunity in action resulting from a detection of sub by ship with direct path sonar, i=1, 2, 3.
PAA(I)	Probability that a type I submarine attacks the ASW inner screen after an unsuccessful attack on the submarine by the screen resulting from a detection with direct path sonar.
PKSC(I)	Probability that a type I submarine kills an ASW inner screen ship in action resulting from detection of sub by ship with direct path sonar.
PSUBAT(I)	Probability that a type I submarine attacks the ASW inner screen following an unsuccessful bottom bounce detection attempt by the screen.
PKOSC(I)	Probability that a type I submarine sinks an ASW inner screen ship without prior detection of the submarine by the screen.
QTU 398	Designator for radiated noise level of enemy anti-screen ship torpedoes. (QTU equal to one implies a quiet torpedo; QTU equal to zero implies a noisy torpedo. No other values are possible).

VM(K)	Conversion factor reflecting change in the ASW inner screen detection probability with direct path sonar in type K sonar conditions because of flaming datum resulting from sinking of screen ship by previously undetected submarine.
CSALVO(I)	Number of anti-shipping torpedoes used by type I submarine.
PDSAU(I,K)	Probability that an (I,K) submarine is detected by the SAV, given the submarine has sunk one or more cargo ships in the convoy.
PSSAU(I,K)	Probability that an (I,K) submarine is sunk by the SAU, given the SAU detected the submarine.
ATTSAU(I)	Does a type I submarine attack a number of the SAU group, given it was not sunk by this group, YES=1.
RASW(I)	Number of times a type I submarine can re-attack the convoy after SAU action.
TORPS(I,ASW)	Number of anti-ASW torpedoes per type I submarine. Note: If only one type of torpedo is used for the play of the game and it is desired to fire this torpedo at all targets, then it should [must] be stored in TORPS (I,ASW).

3.2.1 Evaluation of the Integrals Pr(A)

To evaluate the integrals Pr(A), the equivalences between the first and second Markov chains are exploited to condense the notation. The states E_1, E_2, E_7, and E_8 are seen to be equivalent to the states F_1, F_2, F_5, and F_6 respectively, and therefore, $p_1 = q_1$, $p_2 = q_2$, $p_7 = q_5$ and $p_8 = p_6$. For

$$s = (p_1 + p_2)$$

and

$$f(n) = \frac{1-s^{n+1}}{1-s}$$

and letting D denote the derivative operator with respect to s, the evaluation of the unconditioned probabilities, $P_j^{(n)}$ and $R_j^{(n)}$ yield the following relationships:

$$P_1^{(n)} = p_1 s$$

$$P_2^{(n)} = p_2 s$$

$$P_j^{(n)} = p_j f(n),\ j = 3, \ldots, 8$$

$$R_1^{(n)} = p_1\ p_3\ Ds^n$$

$$R_2^{(n)} = q_2\ p_3\ Df(n)$$

$$R_3^{(n)} = q_3\ p_3\ Df(n)$$

$$R_4^{(n)} = q_4\ p_3\ Df(n)$$

$$R_5^{(n)} = p_7\ p_3\ Df(n)$$

$$R_6^{(n)} = p_8\ p_3\ Df(n)$$

Using these values, equations (3.0, (3.1), (3.2), (3.4) and (3.5) from section 3.1 are evaluated to yield

$$(3.0)' \quad P(A_1) = s^{n+1} + p_5\ f(n)$$

$$(3.1)' \quad P(A_2) = (p_3 + p_4)\ f(n)$$

$$(3.2)' \quad P(A_3) = (p_5 + p_6)\ f(n)$$

$$(3.4)' \quad P(A_4) = (1 + p_3\ D)\ s^{n+1} + (p_5 + p_3 q_3\ D)\ f(n)$$

$$(3.5)' \quad P(A_5) = (q_3 + q_4)\ p_3\ D\ f(n)$$

Generically each of the integrals Pr(A)

$$\Pr(A) = \int_t \Pr(A/g(t))\ \ d\Pr(T \leqq t)$$

is a function of the integral N

$$N = \int_t s^{g(t)}\ d\Pr(T \leqq t)$$

and the first derivative of N with respect to s, assuming the integration and differentiation operations can be interchanged. The distribution $Pr(T \leqq t)$ is non-trivial only for the submarine initially on station and is, in SEALIFT, uniform on the interval (0, ALTIM), i.e., $dPr(T \leqq t) = \frac{dt}{ALTIM}$.

To explore the structure of the function g(t) and remain within the FORTRAN concept of SEALIFT, two new FORTRAN variables are defined

TCON(I) $\overset{d}{=}$ The game at which the I th outgoing convoy crosses the subline,

and

RTCON(I) $\overset{d}{=}$ The game time at which I th returning convoy crosses the subline.

For those submarines initially on station, g(t) is zero for $t < TCON(1)$, and steps to one at t = TCON(1). It then increases by one after each interval TBCON until the first returning convoy crosses the subline, at game time RTCON(1). Until the SEALIFT game parameters have been set, the fine structure of g(t) is not well defined after RTCON(1). It does, however, increase by two every TBCON hours after RTCON(1). Clearly then, the integrals Pr(A) involve extensive summations. To avoid these summations a piecewise continuous linear approximation, h(t), to the step function, g(t), is sought such that for $g(t) \overset{a}{=} h(t)$, the value of N is easily found, viz.,

$$(3.7) \qquad N = \int_t s^{g(t)} \, dt \overset{a}{=} \frac{s^{h(t)}}{Ln(s) \, \frac{dh(t)}{dt}}$$

Incorporating the SEALIFT FORTRAN variables and the fact that in SEALIFT the subline is positioned half way between convoy home port and delivery port it can be seen that

$$TCON(1) = \frac{ONEWAY}{2 \times CONSPD} + CONHR$$

$$RTCON(1) = \frac{3}{2} \times \frac{ONEWAY}{CONSPD} + CONHR + ULTIME$$

and further,

$$TCON(I) = TCON(1) + (I-1)\ TBCON$$

$$RTCON(I) = RTCON(1) + (I-1)\ TBCON$$

If K is defined as the number of outgoing convoys which pass the subline before the first returning convoy passes the subline, then K is the largest integer which satisfies the inequality

$$TCON(K) \leqq RTCON(1)$$

or

$$(K - 1)\ TBCON + TCON(1) \leqq RTCON(1)$$

$$K = \left[\frac{RTCON(1) - TCON(1)}{TBCON}\right] + 1 \quad ,$$

where the brackets denote "the greatest integer less than or equal to." Introducing a new FORTRAN variable H(I) for the expected value analysis, the values of H(I) and the approximation h(t) are defined as:

$$H(1) \overset{d}{=} h(0) = 0$$

$$H(2) \overset{d}{=} h(TCON(1) - 0) = 0$$

$$H(3) \overset{d}{=} h(TCON(1) + 0) = \tfrac{1}{2}$$

$$H(4) \overset{d}{=} h(RTCON(1) - 0) = \tfrac{1}{2} + \frac{1}{TBCON}\ (RTCON(1) - TCON(1))$$

$$H(5) \overset{d}{=} h(RTCON(1) + 0) = K + \tfrac{1}{2}$$

$$H(6) \overset{d}{=} h(ALTIM) = (K + \tfrac{1}{2}) + \frac{2}{TBCON}\ (ALTIM - RTCON(1))$$

Figure 4 depicts both a sample step function g(t), and its linear approximation h(t). Using h(t), the generic integral, N, becomes

$$(3.8) \qquad N = \int_{0}^{TCON(1)} s^{0}dt + \int_{TCON(1)}^{RTCON(1)} s^{h(t)}dt + \int_{RTCON(1)}^{ALTIM} s^{h(t)}dt$$

and, by equation (3.7), becomes

$$= TCON(1) + TBCON \frac{s^{H(4)} - s^{H(3)}}{Ln(s)} + \frac{TBCON}{2} \frac{s^{H(6)} - s^{H(5)}}{Ln(s)}$$

The following derivatives are of immediate interest in evaluating the integrals Pr(A).

$$D\left(\frac{N}{ALTIM}\right) = \frac{TBCON}{ALTIM} \frac{Ln(s)\ (H(4)\ s^{H(4)-1} - H(3)\ s^{H(3)-1}) - (s^{H(4)-1} - s^{H(3)-1})}{Ln^{2}(s)}$$

$$+ \frac{TBCON}{2\ ALTIM} \frac{Ln(s)\ (H(6)\ s^{H(6)-1} - H(5)\ s^{H(5)-1}) - (s^{H(6)-1} - s^{H(5)-1})}{Ln^{2}(s)}$$

and

$$D\left(\frac{ALTIM - N}{ALTIM\ (1 - s)}\right) = \frac{1 - \frac{N}{ALTIM} - (1 - s)\ D\left(\frac{N}{ALTIM}\right)}{(1 - s)^{2}}$$

In terms of these values, the integrals Pr(A) become

(3.9) PA1 $\overset{d}{=}$ Pr (a submarine initially on station survives the first Markov chain to attempt passage home)

$$= \frac{N}{ALTIM} + P_{5} \frac{ALTIM - N}{1 - s}$$

(3.10) PA2 $\overset{d}{=}$ Pr (a submarine initially on station fires one salvo at the cargo ships in the first Markov chain)

$$= (P_{3} + P_{4}) \frac{ALTIM - N}{ALTIM\ (1 - s)}$$

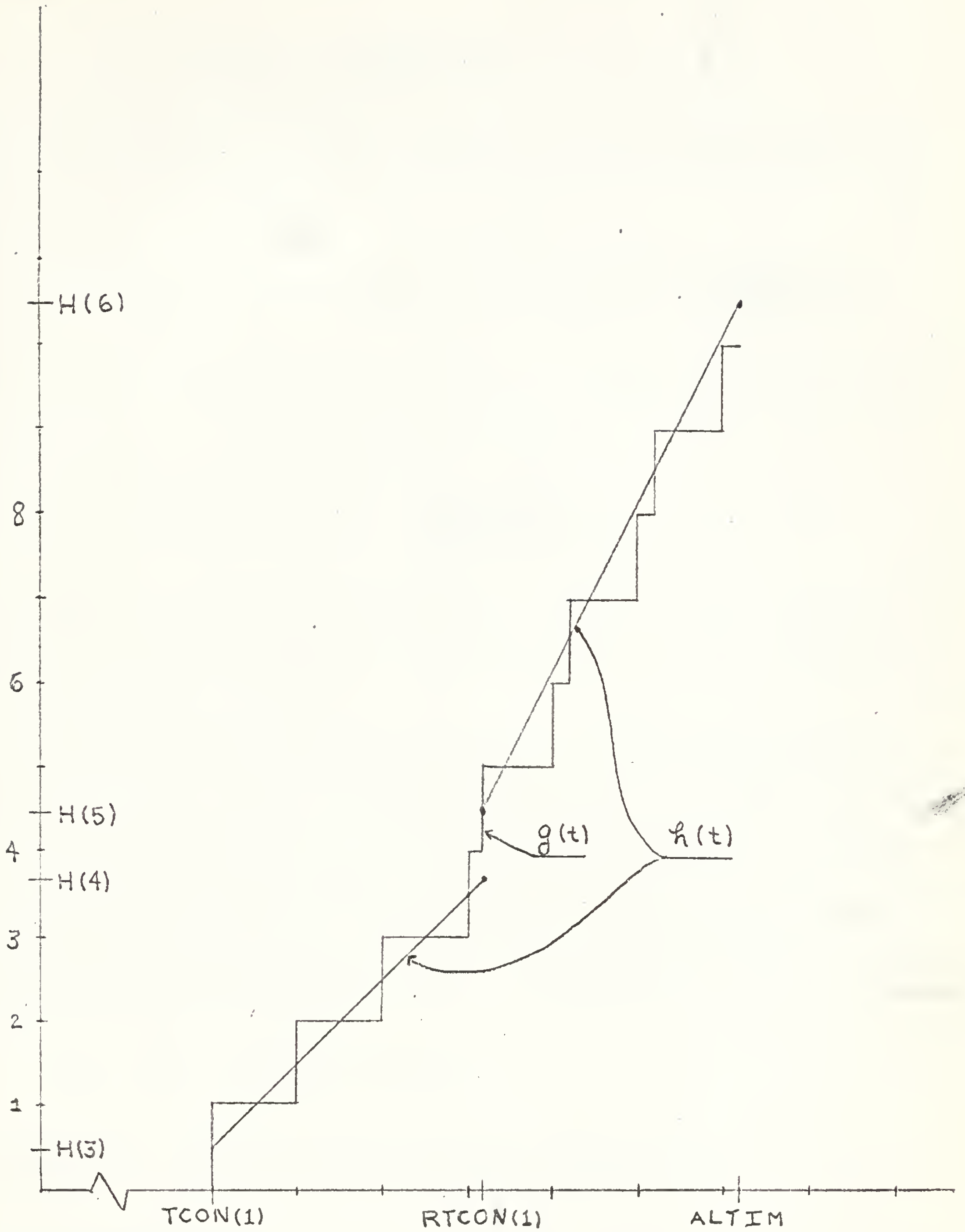

FIGURE 4

A SAMPLE STEP FUNCTION, g(t), AND ITS APPROXIMATION, h(t)

(3.11) PA3 $\stackrel{d}{=}$ Pr (a submarine initially on station fires two salvos at the cargo ships in the first Markov chain)

$$= (p_5 + p_6) \frac{\text{ALTIM} - N}{\text{ALTIM}\ (1 - s)}$$

(3.12) PA4 $\stackrel{d}{=}$ Pr (a submarine initially on station survives the interaction)

$$= (1 + p_3\ D) \frac{N}{\text{ALTIM}} + (p_5 + p_3\ q_3\ D) \frac{\text{ALTIM} - N}{\text{ALTIM}(1 - s)}$$

(3.13) PA5 $\stackrel{d}{=}$ Pr (a submarine initially on station fires on cargo ships in the second Markov chain)

$$= (q_3 + q_4)\ p_3\ D \frac{\text{ALTIM} - N}{\text{ALTIM}\ (1 - s)}$$

This completes the evaluation of the integrals Pr(A) for those submarines initially on station.

The integrals Pr(A) are trivially Stieltjes integrals for those submarines which arrive on station after the war starts. The following is based on the assumption that the arrival time for these submarines is later than RTCON(1), i.e., the full convoy arrival rate of two convoys per TBCON hours has been achieved prior to submarine arrival at the subline. In this case,

$$\Pr(A) = \Pr(A/\ g(\text{maximum value of } t))$$

but

$$g(\max t) = \left[\frac{\text{ALTIM} - \text{submarine transit time to station}}{\text{TBCON}} \times 2 \right],$$

Neglecting possible delays engendered during HUK attacks and during the barrier penetration, the submarine transit time is $\frac{\text{DSUBSTA}}{\text{SUBSPD}}$, therefore

$$Pr(A) = Pr(A/\ n+1 = \left[\frac{\text{ALTIM} \quad \frac{\text{DSUBSTA}}{\text{SUBSPD}}}{\text{TBCON}} \times 2 \right])$$

Letting KONE $\stackrel{d}{=} \left[\frac{\text{ALTIM} - \frac{\text{DSUBSTA}}{\text{SUBSPD}}}{\text{TBCON}} \times 2 \right]$, the following probabilities can be found

(3.14) PA6 $\stackrel{d}{=}$ Pr (a submarine not initially on station survives the first Markov chain to attempt passage home)

$$= s^{\text{KONE}} + p_5 \frac{1 - s^{\text{KONE}}}{1 - s}$$

(3.15) PA7 $\stackrel{d}{=}$ Pr (a submarine not initially on station fires one salvo at the cargo ships in the first Markov chain)

$$= (p_3 + p_4) \frac{1 - s^{\text{KONE}}}{1 - s}$$

(3.16) PA8 $\stackrel{d}{=}$ Pr (a submarine not initially on station fires two salvos at the cargo ships in the first Markov chain)

$$= (p_5 + p_6) \frac{1 - s^{\text{KONE}}}{1 - s}$$

(3.17) PA9 $\stackrel{d}{=}$ Pr (a submarine not initially on station survives the submarine-convoy interaction)

$$= (1 + p_3\ D)\ s^{\text{KONE}} + (p_5 + p_3\ q_3\ D) \frac{1 - s^{\text{KONE}}}{1 - s}$$

(3.18) PA10 $\stackrel{d}{=}$ Pr (a submarine not initially on station fires on cargo ships in the second Markov chain)

$$= (q_3 + q_4)\ p_3\ D \frac{1 - s^{\text{KONE}}}{1 - s}$$

This completes the evaluation of the integrals in terms of the evaluated or

known FORTRAN variables and the as yet unevaluated transition probabilities.

3.2.2 The Barrier Penetration, Transition Probabilities and Related Expected Numbers

Until now, it has been convenient to suppress in the analysis any notation concerning submarine type and sonar condition. The Markov chain notation was difficult enough without the burden of two additional subscripts. The remainder of this analysis will distinguish submarines by type and by assigned sonar condition. It will be seen that each of the probabilities given by equations (3.9) through (3.18) is a function of submarine type and sonar condition.

<u>Placement of submarines</u>. In SEALIFT, there are eight ways to classify submarines of a particular type by placement at the start of war: two choices for on station or not and four choices of sonar box. This classification is done by Monte-Carlo methods. Type sonar condition within each box is determined by input. For the purposes of this analysis, sonar box classification is immaterial except for the restriction that the fourth sonar box is not included in the analysis. For the initial placement of submarines the expected number of type I submarines initially on station in type K sonar conditions is given by

$$E2(I,K) \;=\; NSUB'I'T \times FSUB'I'OS \times \sum P'I'BOX(J)$$

where the summation is over all sonar boxes assigned sonar condition K. The expected number of type I submarines not initially on station but assigned to type K sonar conditions, E3(I,K), is derived from two sources, the fraction of on hand submarines not initially on station and the submarines built during the war. Using a nominal thirty day month,

$$E3(I,K) \;=\; (NSUB'I'T\ (1 - FSUB'I'OS) + NSUB'I' \times \frac{TWAR}{720}) \times \sum P'I'BOX(J)$$

where the summation is over all sonar boxes assigned sonar condition K.

Barrier penetration. A submarine will successfully transit the eight distinct barriers if and only if she successfully transits each barrier. In the SEALIFT game, the barriers are evenly divided between two types, constant and variable. The latter type are variable in the sense that the barrier-detects-submarine probability will vary with barrier strength. Under the basic assumptions used in this expected value analysis, however, each of the barriers is treated as a constant barrier. For constant barriers therefore

$$\begin{aligned} \text{PA11(I,J)} &\overset{d}{=} \text{Pr (a type I submarine successfully transits the J th constant barrier)} \\ &= 1 - \text{Pr (she is sunk in this barrier)} \\ &= 1 - \text{PBARSC(I,J)} \end{aligned}$$

and

$$\begin{aligned} \text{PA12(I,J)} &\overset{d}{=} \text{Pr (a type I submarine successfully transits the J th variable barrier)} \\ &= 1 - \text{Pr (she is sink in this barrier)} \\ &= 1 - \text{PBARSV(I,J)} \times (\text{PVBDET(I,J)} + \text{VAIRDET(J)}) \end{aligned}$$

Therefore the probability that a type I submarine successfully transits the barriers, PO(I), is

$$\text{PO(I)} = \prod_{J=1}^{4} \text{PA11(I,J)} \times \text{PA12(I,J)}$$

Convoy detection. In the SEALIFT game, all convoys cross the subline at the subline's midpoint. Detection probabilities are of the "cookie cutter" type where submarines are placed (at each convoy crossing) uniformly on the subline. The probability an (I,K) submarine detects a convoy on any

one trial, 1 - P1(I,K), is then

$$1 - P1(I,K) = \frac{2 \times RGDT(K,I)}{SUBLINE}$$

The transition probabilities, $p_1 = q_1 = P1(I,K)$, are thereby fully defined, and are dependent on submarine type and on sonar condition.

<u>Penetration of the forward screen escort</u>. This event and the following events are the substance of Event 39 (Submarine versus convoy) of the SEALIFT game. Figure 5 is the simplified truth table (tree) of that part of Event 19 from which the FSE associated probabilities and expectations used in this analysis are derived. All of these probabilities are conditioned on the submarine having detected a convoy. The input parameters FAIRSC and ATSFE(I) are obviously not probabilities. FAIRSC is transformed into a 0,1 parameter in this analysis by the transformation 2 - FAIRSC, allowing its use in probability statements. ATSFE(I) is another tactical limitation of the type I submarine and this parameter is usually a "small" integer for the conventional submarine and a "large" integer for the nuclear submarine. If ATSFE(I) = 0, a type I submarine will not be allowed to attempt repenetration of the FSE in the event she is rebuffed. For this submarine in type K sonar conditions the following probabilities are obtained

$$\begin{aligned} P7(I,K) &\overset{d}{=} \text{Pr (a submarine penetrates the FSE; ATSFE(I) = 0)} \\ &= 1 - (PDFS(I,K) + PDFER) \end{aligned}$$

$$\begin{aligned} P9(I,K) &\overset{d}{=} \text{Pr (a submarine returns to station; ATSFE(I) = 0)} \\ &= (PDFS(I,K) + PDFER) \times (1 - PDFCE(I,K)) \times (1 - (2 - FAIRSC) \\ &\quad \times PSFSE(I) \times PFCES(I,K) \times PSFCE(I,K)) \end{aligned}$$

and

$$\begin{aligned} P8(I,K) &\overset{d}{=} \text{Pr (a submarine is killed attempting FSE penetration; ATSFE(I)=0)} \\ &= 1 - P9(I,K) - P7(I,K) \end{aligned}$$

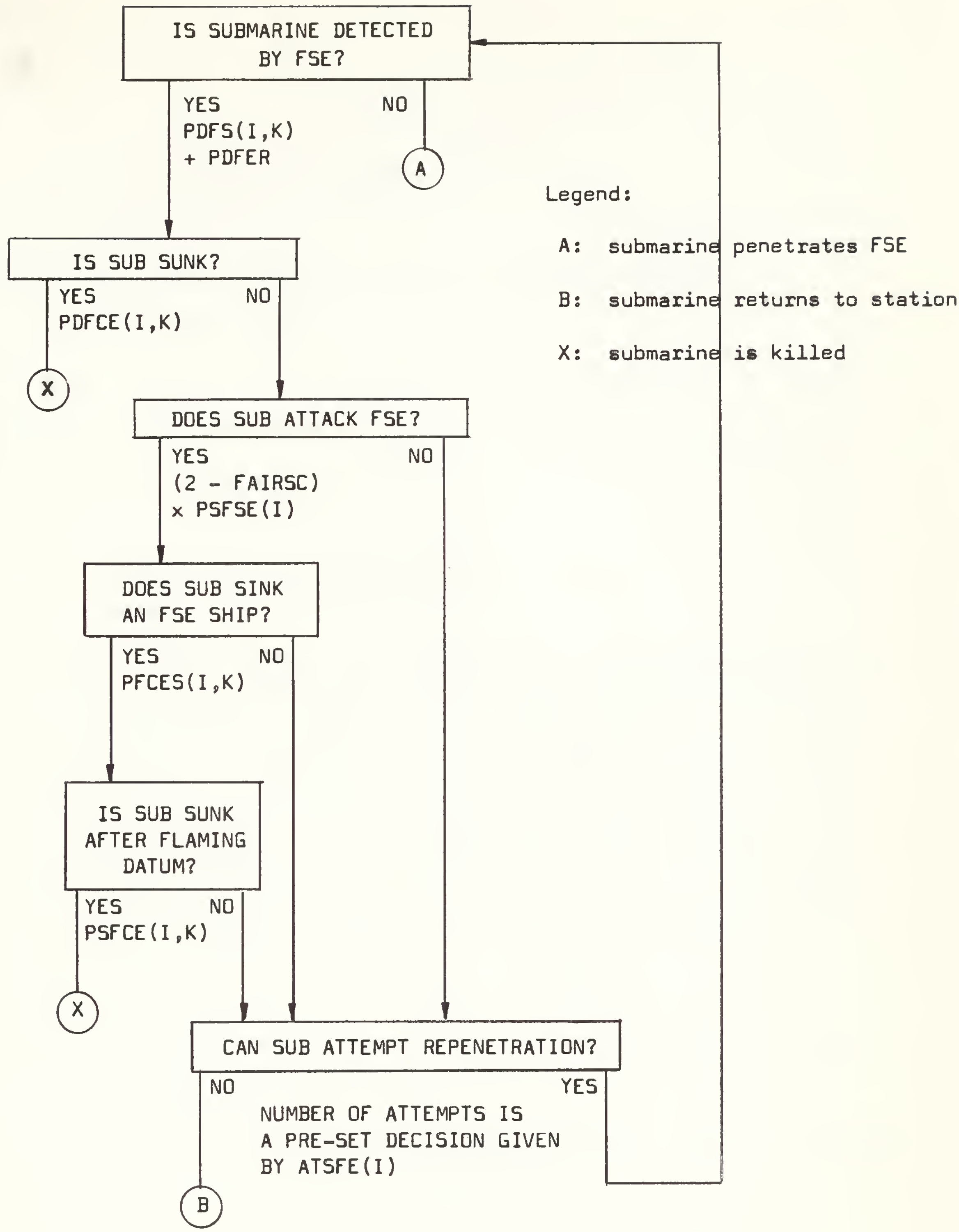

FIGURE 5

ARBORESCENCE ILLUSTRATING THE SUBMARINE-FORWARD SCREEN ESCORT INTERACTION

In general, if ATSFE(I) = n, n attempts at penetration will be allowed, and

$$P7(I,K,n) \stackrel{d}{=} \text{Pr (a submarine penetrates the FSE; ATSFE(I) = n)}$$

$$= P7(I,K) \quad \frac{1 - (P9(I,K))^{n+1}}{1 - P9(I,K)}$$

$$P9(I,K,n) \stackrel{d}{=} \text{Pr (a submarine returns to station; ATSFE(I) = n)}$$

$$= (P9(I,K))^{n+1}$$

and

$$P8(I,K,n) \stackrel{d}{=} \text{Pr (a submarine is killed attempting FSE penetration; ATSFE(I) = n)}$$

$$= P8(I,K) \quad \frac{1 - (P9(I,K))^{n+1}}{1 - P9(I,K)}$$

As derived herein these values are predicated only on the approximation that the FSE-detects-submarine probability has not been degraded by prior submarine attack either by the submarine under consideration (ATSFE(I) $>$ 0) or by earlier submarines and that when the submarine attempts to attack the screen she <u>will</u> have sufficient torpedoes to do so. To find the expected number of torpedoes a submarine fires at FSE ships given she penetrates the screen, suppose ATSFE(I) = n, and define the following events

C_j = event that a submarine penetrates the screen on the j th try, j = 1, ..., n+1

D_i = event that a submarine fires i salvos, each of size ASALVO(I) during an attempted penetration, i = 0, ..., n+1

Then E1(I,K) can be defined as the expected number of torpedoes fired by an (I,K) submarine during an attempted penetration of the FSE given she penetrates the FSE and ATSFE(I) = n, i.e.,

(3.19)

$$E1(I,K) = ASALVO(I) \sum_{i=0}^{n} i \frac{\sum_{j=i+1}^{n+1} Pr(C_j, D_i)}{P7(I,K,n)}$$

The probability Pr (C_j, D_i) can be derived by the following argument: the event (C_j, D_i) is a sequence of j trials where the j th and last trial is a non-detection by the FSE. This trial is preceeded by i trials (not necessarily in sequence) in which the submarine is detected, not sunk, does attack the FSE, and either sinks an FSE ship and is not killed or does not sink an FSE ship. The last trial is also preceeded by j - i - 1 trials in which the submarine is detected, not sunk, and does not attack the FSE. Denote the probability of each type trial, in the order listed, by T1, T2, and T3. Then

$$T1(I,K) = P7(I,K)$$

$$T2(I,K) = (PDFS(I,K) + PDFER) \times (1 - PDFCE(I,K)) \times (2 - FAIRSC) \times PSFSE(I) \times (1 - PFCES(I,K) \times PSFCE(I,K))$$

and

$$T3(I,K) = (PDFS(I,K) + PDFER) \times (1 - PDFCE(I,K)) \times (1 - (2 - FAIRSC) \times PSFSE(I))$$

Therefore,

$$Pr(C_j, D_i) = \binom{j-1}{i} \times (T2(I,K))^i \times (T3(I,K))^{j-i-1} \times P7(I,K)$$

and equation (3.19) becomes

(3.19)'

$$E1(I,K) = ASALVO(I) \sum_{i=1}^{n} i \frac{\sum_{j=i+1}^{n+1} \binom{j-1}{i} (T2(I,K))^i \times (T3(I,K))^{j-i-1}}{\frac{1 - (P9(I,K))^{n+1}}{1 - P9(I,K)}}$$

Penetration of the inner screen (ASW). Having penetrated the FSE, an attacking submarine must penetrate the inner screen prior to reaching the cargo ships. Figure 6 is the simplified tree of that part of Event 19 (Submarine versus convoy) of SEALIFT from which the ASW associated probability of penetration is derived for this expected value analysis. This portion of the analysis is based upon an undated and informal modification of Event 19 which, however, modifies only the ASW penetration portion. The assumptions upon which the tree is constructed are the same as used for the FSE penetration analysis. That is, the ASW-detects-submarine probability is assumed not degraded by prior ASW ship losses and that if a submarine attacks the ASW, she is assumed to have sufficient torpedoes to do so. Because of the method used in deriving the ASW penetration probability for this analysis, parasitic torpedo usage could not be determined. It was observed that two mutually exclusive events prevailed, either the submarine penetrated the ASW or she was sunk. The method used for computing the probability of these events occurring was to first determine the probability the submarine was sunk. The desired probability of penetration is found by complementation.

The various paths to "submarine death" during the ASW penetration are stylized in Figure 7. The circled integers, I, and the circled arabic A, A, in Figure 7 represent branch points corresponding to those of the tree, Figure 6. The circled arabic X, X, represents the event "submarine is killed." The symbol p^i_{rs} is the probability of transition from branch point r to branch point s, indexed by the integer i. From Figures 6 and 7 the probabilities for an (I,K) submarine are read off:

$$p_{14} = \mathrm{PDETU(I,K)} \times \mathrm{VA(K)}$$

$$p_{A8} = \mathrm{PBB(I,K)} \times \mathrm{PAT(I,K)}$$

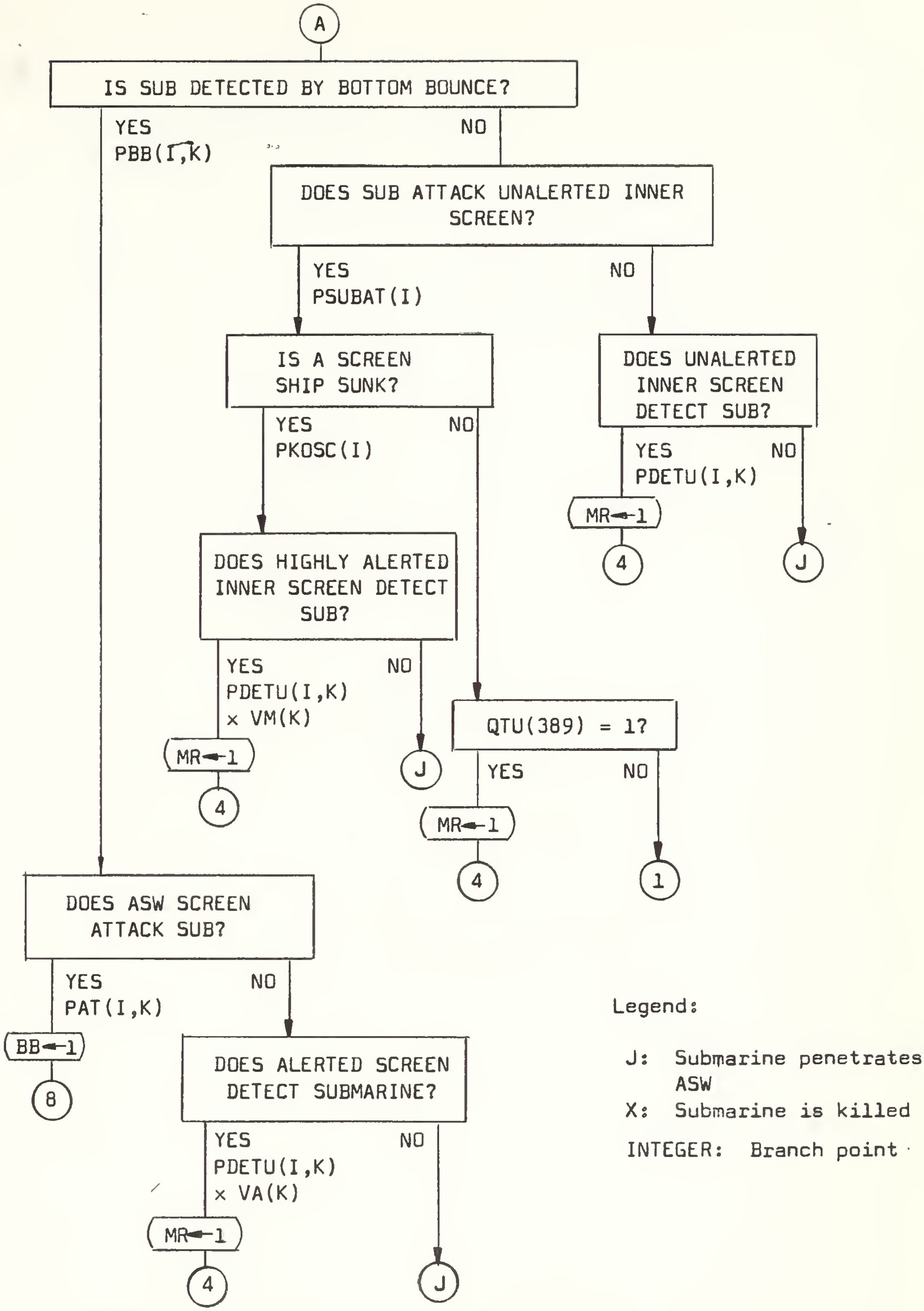

FIGURE 6

ARBORESCENCE ILLUSTRATING THE SUBMARINE-ASW SCREEN INTERACTION

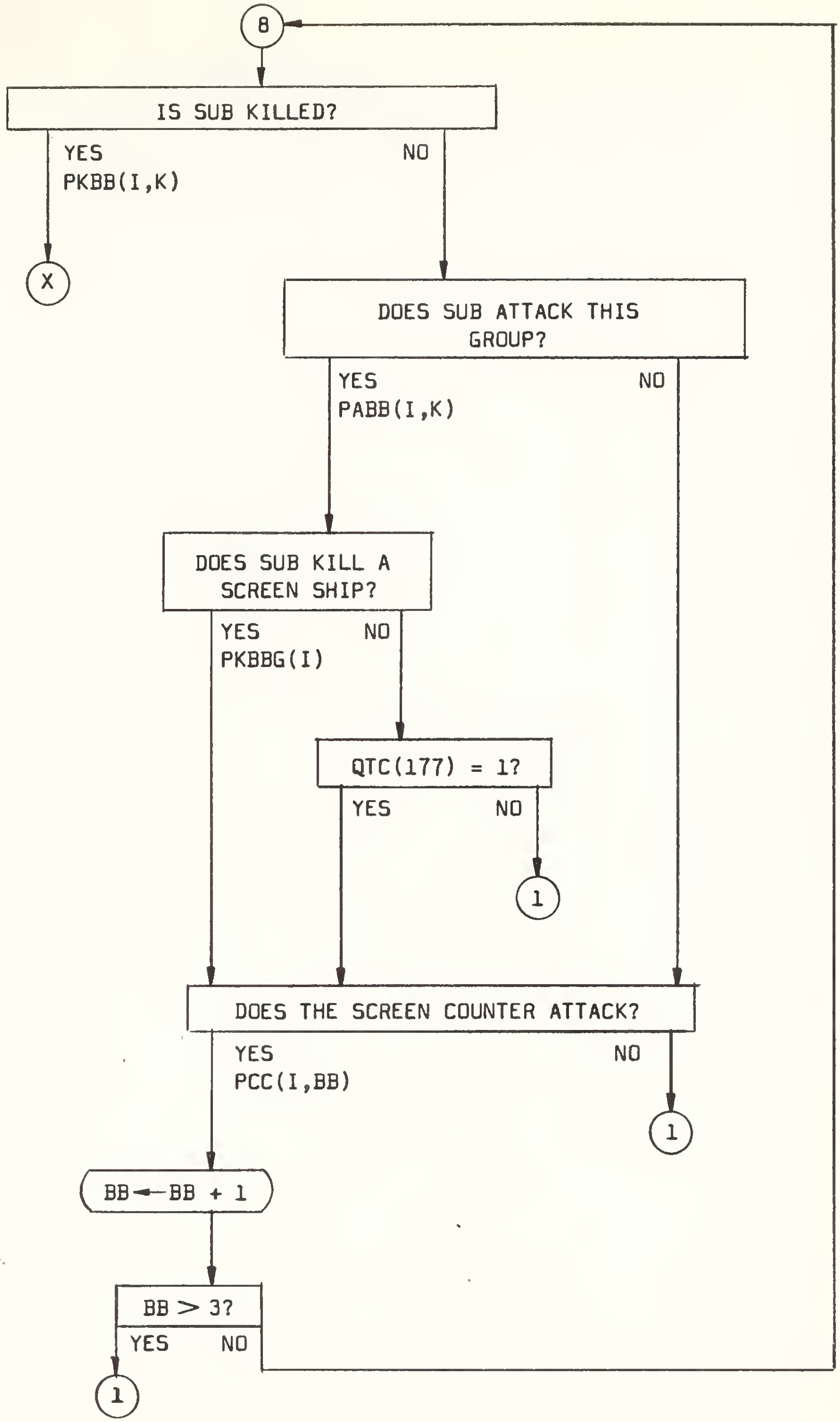

FIGURE 6 (continued)

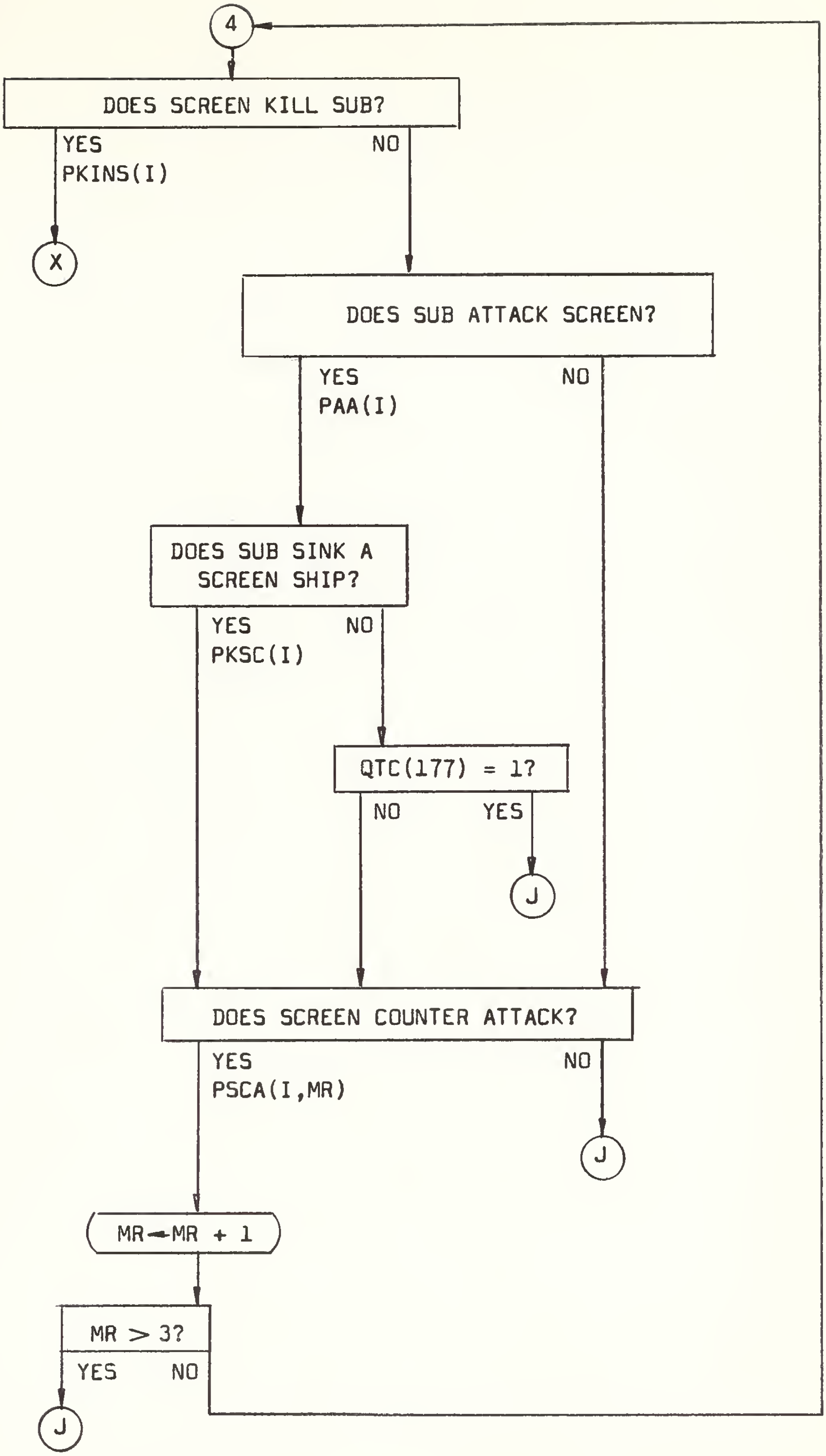

FIGURE 6 (continued)

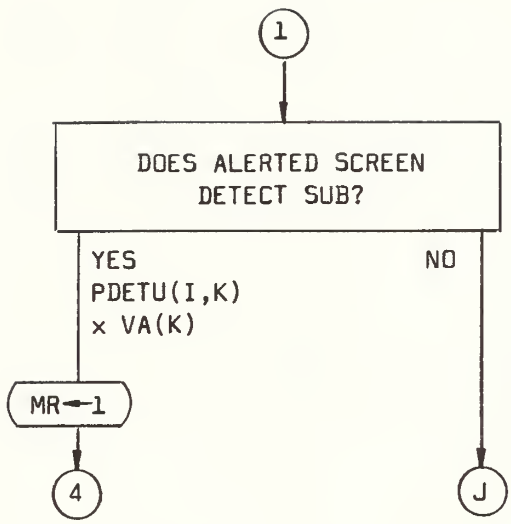

FIGURE 6 (continued)

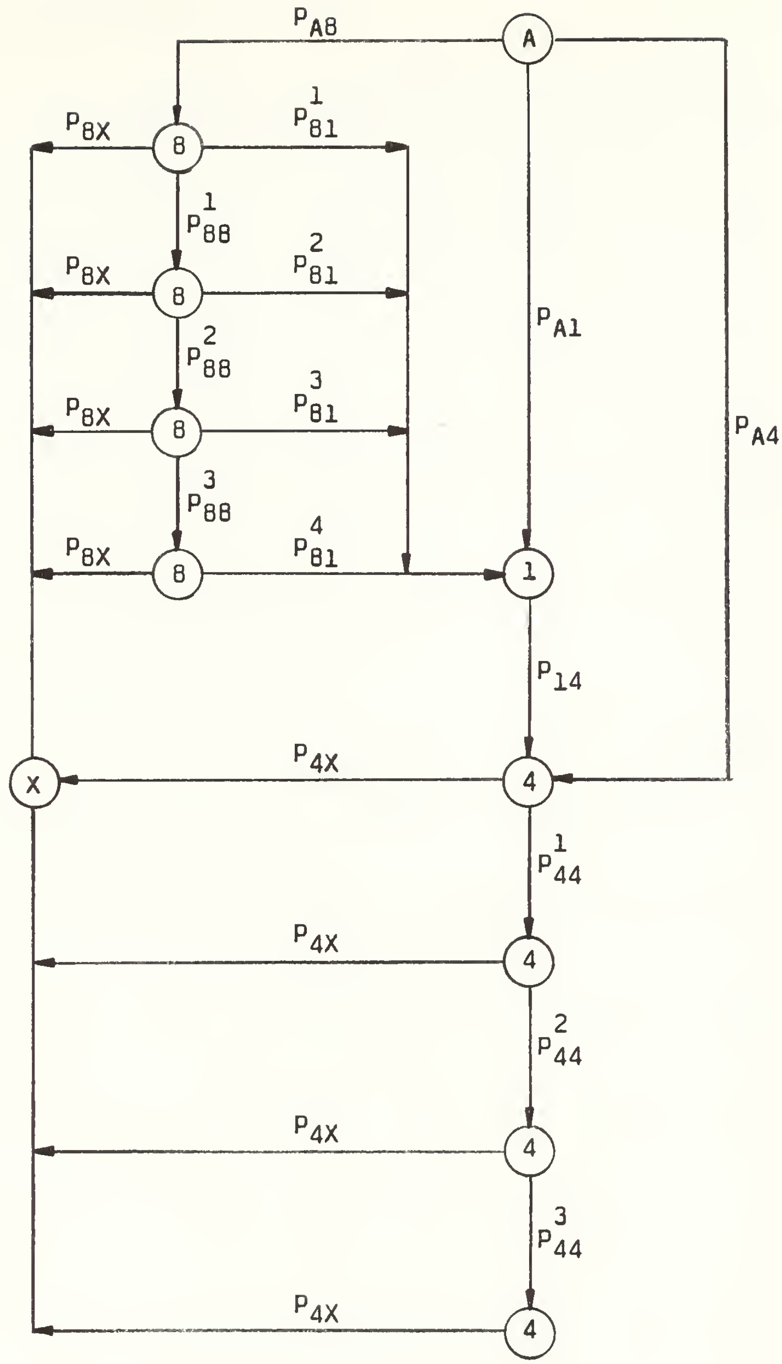

FIGURE 7

STYLIZED PATHS TO SUBMARINE DEATH DURING THE SUBMARINE-ASW SCREEN INTERACTION

$$P_{8X} = PKBB(I,K)$$

$$P^{i}_{88} = (1 - PKBB(I,K)) \times PCC(I,i) \times (1 - PABB(I,K) + PABB(I,K) \times$$

$$(PKBBG(I) + (1 - PKBBG(I)) \times (QTC(177)))), \quad i=1, \ldots, 3$$

$$P^{i}_{81} = 1 - P^{i}_{88} - P^{i}_{8X}, \quad i=1, \ldots, 3$$

$$P^{4}_{81} = 1 - PKBB(I,K)$$

$$P^{i}_{44} = (1 - PKINS(I)) \times PSCA(I,i) \times (1 - PAA(I) \times (PKSC(I)$$

$$+ (1 - PKSC(I)) \times (1 - QTC(177)))), \quad i=1, \ldots, 3$$

$$P_{4X} = PKINS(I)$$

$$P_{A1} = (1 - PBB(I,K)) \times PSUBAT(I) \times (1 - PKOSC(I)) \times (1 - QTU(389))$$

and

$$P_{A4} = PBB(I,K) \times (1 - PAT(I,K)) \times PDETU(I,K) \times VA(K)$$

$$+ (1 - PBB(I,K)) \times (PSUBAT(I) \times (PKOSC(I) \times PDETU(I,K) \times VM(K)$$

$$+ (1 - PKOSC(I)) \times QTU(389)) + (1 - PSUBAT(I)) \times PDETU(I,K)))$$

The probability that the submarine penetrates the inner screen, P3(I,K), can then be found as

P3(I,K) = 1 - Pr (Submarine is sunk attempting ASW penetration)

= 1 - Pr (a transit from the first (4) to (X) by all possible paths) x Pr (a transit from (A) to first (4) by all possible paths) + Pr (a transit from (A) to (X) by all possible paths not passing through any (4))

Each of these probabilities can be read from Figure 8 to be the following:

Pr (a transit from the first (4) to (X) by all possible paths)

$$= P_{4X} + P_{44}^{1} (P_{4X} + P_{44}^{2} (P_{4X} + P_{44}^{3} P_{4X}))$$

Pr (a transit from (A) to first (4) by all possible paths)

$$= P_{A1} P_{14} + P_{A8}(P_{81}^{1} P_{14} + P_{88}^{1}(P_{81}^{2} P_{14} + P_{88}^{2}(P_{81}^{3} P_{14} + P_{88}^{3} P_{81}^{4} P_{14})))$$

and

Pr (a transit from (A) to (X) by all possible paths not passing through any (4))

$$= P_{A8} (P_{8X} + P_{88}^{1}(P_{8X} + P_{88}^{2}(P_{8X} + P_{88}^{3} P_{8X})))$$

<u>Convoy attack phase</u>. It is particularly in the convoy attack that the flexibility of the SEALIFT game complicates the expected value analysis. Possible realizations of this phase depend heavily upon the chosen war option as listed in SEALIFT I [1] and upon the various torpedo salvo sizes chosen for each submarine type. In view of these considerations and the problem which prompted this expected value analysis, a particular characterization of the convoy attack phase has been chosen. It is assumed that

1. one type of torpedo will be used against both cargo and screen ships,
2. the salvo size against cargo ships is CSALVO(I) and that CSALVO(I) $\stackrel{a}{=} \frac{1}{2}$ the initial torpedo supply,
3. the salvo size against screen ships is ASALVO(I) and that ASALVO(I) $\stackrel{a}{=}$ 1/6 x CSALVO(I),
4. in firing one CSALVO against cargo ships, the combination of CSALVO(I) and the probability of kill per torpedo

essentially eliminates the probability of sinking no cargo ships, and

5. having fired two salvos at cargo ships, the submarine's torpedo supply is exhausted.

Based on these assumptions, Figure 8 is the simplified tree of that part of Event 19 (Submarine versus convoy) of SEALIFT from which the convoy attack associated probabilities are derived from the first Markov chain in this analysis. For an (I,K) submarine sonar conditions the conditional probabilities of interest are

P41(I,K) = Pr (a submarine shoots one salvo at cargo ships in the first Markov chain)

= PDSAU(I,K)

P42(I,K) = Pr (a submarine shoots two salvos at cargo ships in the first Markov chain)

= 1 - PDSAU(I,K)

P5(I,K) = Pr (a submarine escapes/ she shoots one salvo at cargo ships in the first Markov chain)

= 1 - PSSAU(I,K)

and

P6(I,K) = Pr (a submarine escapes/ she shoots two salvos at cargo ships in the first Markov chain)

= 1 - PDSAU(I,K) x PSSAU(I,K)

If the expected number of torpedoes fired by an (I,K) submarine at cargo ships in the first Markov chain given the submarine fires one and two salvos are E4(I,K) and E5(I,K) respectively, then E4 and E5 are roughly approximated by:

E4(I,K) $\cong$ CSALVO(I)

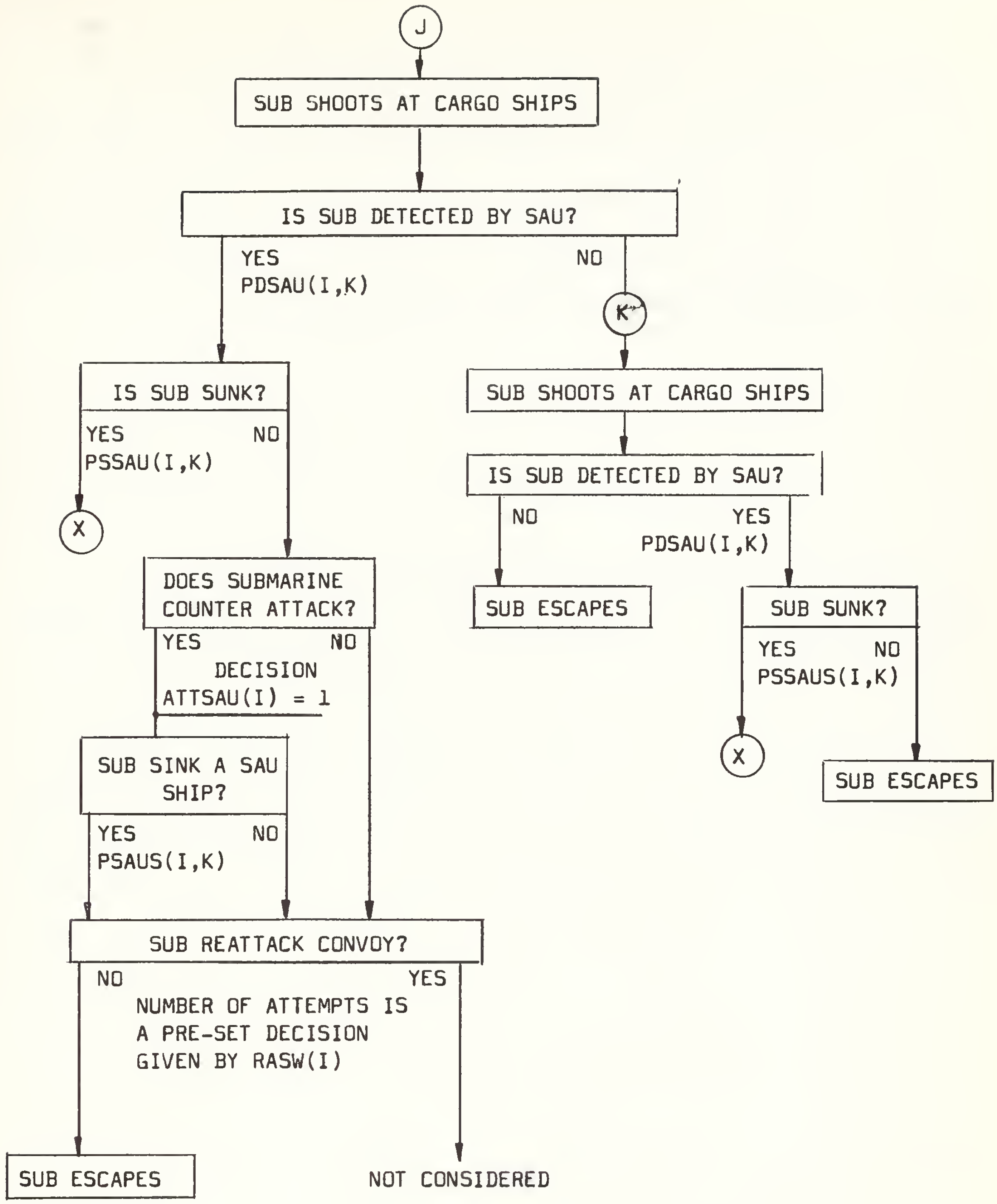

FIGURE 8

ARBORESCENCE ILLUSTRATING THE CARGO SHIP ATTACK PHASE

and

$$E5(I,K) \overset{a}{=} TORPS(I,ASW) - CSALVO(I) - E1(I,K)$$

The tree associated with the cargo attack phase for those submarines which successfully enter the second Markov chain is obtained from Figure 9 by changing the entire right branch at the branch point K into the event "Submarine Escapes." The conditional probabilities of interest are then

$$P4(I,K) = \Pr(\text{a submarine shoots her remaining torpedoes at cargo ships})$$

$$= 1$$

$$P11(I,K) = \Pr(\text{a submarine escapes given she shoots})$$

$$= P5(I,K)$$

The expected number of torpedoes a submarine fires at cargo ships in the second Markov chain given she fires, E6(I,K), is roughly approximated by

$$E6(I,K) \overset{a}{=} TORPS(I,ASW) - CSALVO(I) - 2 \times E1(I,K)$$

<u>The transition probabilities</u>. The calculation of the Markov chain transition probabilities needed for this expected value analysis can now be obtained by combining the conditional probabilities derived in the previous sections. These transition probabilities are:

$$p_1 = q_1 = P1(I,K)$$

$$p_2 = q_2 = (1 - P1(I,K)) \times P9(I,K,n)$$

$$p_3 = (1 - P1(I,K)) \times P7(I,K,n) \times P3(I,K) \times P41(I,K) \times P5(I,K)$$

$$p_4 = (1 - P1(I,K)) \times P7(I,K,n) \times P3(I,K) \times (1 - P5(I,K))$$

$$p_5 = (1 - P1(I,K)) \times P7(I,K,n) \times P3(I,K) \times P42(I,K) \times P6(I,K)$$

$$p_6 = (1 - P1(I,K)) \times P7(I,K,n) \times P3(I,K) \times P42(I,K) \times (1 - P6(I,K))$$

$$p_7 = q_5 = (1 - P1(I,K)) \times P7(I,K,n) \times (1 - P3(I,K))$$

$$p_8 = q_6 = (1 - P1(I,K)) \times P8(I,K,n)$$

$$q_3 = (1 - P1(I,K)) \times P7(I,K,n) \times P3(I,K) \times P4(I,K) \times P5(I,K)$$

$$q_4 = (1 - P1(I,K)) \times P7(I,K,n) \times P3(I,K) \times P4(I,K) \times (1 - P5(I,K))$$

3.2.3 Expected Number of Torpedoes Fired at Cargo Ships

In order to determine the expected number of torpedoes fired at cargo ships for any given cycle, define the following to be

$F(u,v,I,K) \stackrel{d}{=}$ event an (I,K) submarine fires u salvos at cargo ships in the v th Markov chain, $u, v = 1, 2$

$NS(w,I,K) \stackrel{d}{=}$ the number of (I,K) submarines which arrive on station for the w th cycle, $w = 1, 2, 3, \ldots$

$NT(w,I,K) \stackrel{d}{=}$ the total number of torpedoes fired at cargo ships during the w th cycle by (I,K) submarines, $w = 2, 3, \ldots$

The following equation is then used

(3.20)

$$\begin{aligned} E(NT(w,I,K)) = E(NS(w,I,K)) \times \Big(& Pr(F(1,1,I,K)) \times E4(I,K) \\ & + Pr(F(2,1,I,K)) \times E5(I,K) \\ & + Pr(F(1,2,I,K)) \times E6(I,K)\Big) \end{aligned}$$

For the first cycle, distinction is made for the initially on-station, initially not-on-station contingency. For each submarine type and for each sonar condition the expected number of torpedoes fired in each contingency is computed as

$E(NT_1(1,I,K)) \stackrel{d}{=}$ E (number of torpedoes fired by (I,K) submarine initially on station during the first cycle)

$$= E2(I,K) \times \Big(PA2(I,K) \times E4(I,K) + PA3(I,K) \times E5(I,K) + PA5(I,K) \times E6(I,K) \Big)$$

and

$$E(NT_2(1,I,K)) \overset{d}{=} E\ (\text{number of torpedoes fired by } (I,K) \text{ submarines initially not-on-station during the first cycle})$$

$$= E3(I,K) \times PO(I,K) \times \Big(PA7(I,K) \times E4(I,K) + PA8(I,K) \times E5(I,K) + PA10(I,K) \times E6(I,K) \Big)$$

The expected number of survivors from the first cycle, i.e., those who safely reach home port, EC(1,I,K), is

$$EC(1,I,K) = (E2(I,K) \times PA4(I,K) + E3(I,K) \times PO(I,K) \times PA9(I,K)) \times PO(I,K)$$

According to the rules of SEALIFT, the survivors are re-apportioned among the sonar boxes to maintain balance. The expected number of re-apportioned survivors of the (I,K) classification, EC(2,I,K), is

$$EC(2,I,K) = EC(1,I,K) \times \sum P'I'BOX(J)$$

where the sum is over those sonar boxes of like sonar conditions. To determine E(NT(2,I,K)), equation (3.20) is applied again, and becomes

$$E(NT(2,I,K)) = \frac{EC(2,I,K)}{E3(I,K)} \times E(NT_2(1,I,K))$$

The expected number of survivors from the second cycle, EC(3,I,K), is

$$EC(3,I,K) = EC(2,I,K) \times PA9(I,K) \times PO(I,K) \times PO(I,K)$$

For the w th cycle, $w \geqq 3$, the expected number of re-apportioned survivors

from the previous cycle, EC(2w-2,I,K), serve as the submarine input, and the following iteration formulae are found,

$$EC(2w-2,I,K) = EC(2w-3,I,K) \times \frac{EC(2w-4,I,K)}{EC(2w-5,I,K)}$$

and

$$E(NT(w,I,K)) = EC(2w-2,I,K) \times \frac{E(NT(w-1,I,K))}{EC(2w-4,I,K)}$$

The expected number of survivors from the w th cycle (not yet re-apportioned among the sonar boxes), EC(2w-1,I,K) is

$$EC(2w-1,I,K) = EC(2w-2,I,K) \times \frac{EC(2w-3,I,K)}{EC(2w-4,I,K)}$$

An estimate of the number of cycles for the war is given by the minimum number of cycles a live submarine, not initially on station, might expect to achieve. This number is a function of submarine type and is given by the integer L(I)

$$L(I) = \left[\frac{TWAR}{ALTIM(I) + DTSPRT(I) + \frac{DSUBSTA}{SUBSPD(I)}} \right]$$

An estimate for total expected number of torpedoes fired at cargo ships by (I,K) submarines for the war, E(NT(I,K)), is then

$$E(NT(I,K)) = E(NT_1(1,I,K)) + E(NT_2(1,I,K)) + \sum_{w=2}^{L(I)} E(NT(w,I,K)$$

Multiplying this number by the single shot kill probability, PSCS(I,K), provides an estimate for the number of cargo ships sunk by (I,K) submarines, E(CS(I,K)),

$$E(CS(I,K)) = E(NT(I,K)) \times PSCS(I,K)$$

Finally, summing E(CS(I,K)) over the indicies I and K provides an estimate for the total expected number of cargo ships sunk during the entire war, E(CS)

$$E(CS) = \sum_{I=1}^{3} \sum_{K=1}^{2} E(CS(I,K)).$$

4. Conclusions and Recommendations

As a model purporting to represent a probabilistic real world situation, the SEALIFT game, the analysis presented in this thesis remains largely untested. The opportunity and data base for testing the model's reliability, accuracy and sensitivity to the proposed approximations are, however, readily available. If the SEALIFT game is contemplated as an analytical tool in the CYCLOPS III study, the accuracy of this expected value model should be explored. An acceptably reliable programmed version of this model should effect appreciable savings in computer time.

For her continued patience, encouragement and good humor, I acknowledge the support of my wife in the preparation of this thesis.

BIBLIOGRAPHY

1. Naval Warfare Analysis Group, The Center for Naval Analyses. An event store computer program for determining sealift capabilities and attrition in an ASW environment, CNA Computer Program 58-63P: SEALIFT I, by M. Riess and G.A. Westlund. February 1964. Research Contribution No. 42.

APPENDIX I

DETAILED DESCRIPTION OF THE SEALIFT GAME

The following description is quoted from SEALIFT I [1] :

II. GENERAL DESCRIPTION

Convoys

In this game, friendly forces have two ports to transit between home port and delivery port. During this transit the convoy may cross up to two "constant" barriers (oneway transit). The constant barrier in this simulation can sink ships that cross it but cannot itself be weakened by these sinkings or by the actions of the ships crossing it. Mine fields, air barriers, etc., can be described as "constant" barriers. The "variable" barrier, used against submarines, can sink ships that traverse it but can be weakened by having the ships that comprise it sunk. Barriers of this type might be submarine barriers, surface ship barriers, etc. The "variable" type of barrier will be described in greater detail later in this section. It should be pointed out that while each barrier that could be encountered by a convoy or submarine will be described herein, any one or all of these barriers may be removed for a desired play simply by setting to zero its probability of detecting a prospective "transitor"...

At certain times, which are inputs, a convoy is formed in the home port. The convoy is made up of three types of ships: cargo ships, ASW escort ships, and forward screen ships*. Each convoy when it forms draws its required complement of ships from a "port pool." This pool is stocked by ships from returning convoys and by the ship building rate. If the required ships are available, the convoy sails toward its delivery port: if not, the game is stopped, and a "printout" is made of the game results to date including the event that caused the stoppage.

At one hundred miles from its home port, all ships of the convoy except the forward screen escorts pass through a constant barrier (barrier 1C). The losses, if any, are recorded and subtracted from the convoy.

After having transited the first constant barrier, the convoy becomes liable to an air attack. This attack is played as the probability of a convoy ship being sunk per day at sea by air attack. The convoy then enters the submarine patrol area and may encounter a submarine while in this area. The submarine/convoy encounter will be described in greater detail later.

Ships lost because of any one of the aforementioned reasons are noted by type of ship sunk, method of sinking, and time of loss, and are subtracted from the convoy to be printed out at the close of the simulation. If these

* Variations in inputs permit the forward screen to be composed alternately of aircraft, sonobuoys, submarines, etc.

losses include ASW escorts or forward screen escorts, the convoy's ability to detect** enemy submarines will be diminished. Each time ASW escorts and forward screen escorts are sunk, new lower detection probabilities for the various parts of the convoy are calculated.

After passing through the submarine patrol area, the convoy becomes liable to another air attack, then passes through a second constant barrier one hundred miles from its delivery port. After reaching the delivery port, the convoy unloads and stays in port a prescribed number of days. During this period the cargo ships are again liable to air attack. Of those ships sunk in an air attack, one-half are assumed to be sunk before unloading and one-half after unloading. The number of cargo ships delivered are now tabulated and the convoy is ready to proceed home.

Proceeding home, the convoy follows the same route in reverse order, passing through the same barriers, the same submarine patrol area, and the same series of air attacks. When it reaches its home port, the number of ships remaining in the convoy are placed in a "ship pool" by ship type. These ship pools will then be used to make up future convoys.

Submarines

This model is designed to handle three types of submarines. All of the parameters which describe the submarine types are inputs to the game. At the start of the war, the opposition submarines, chosen by type of submarine as well as by percentage of each type, are placed in two predetermined "game positions," selected according to the option of the user. The two game positions are "on station" and "in port." For those submarines that start the game in port, a given number (input) will leave at a given time interval (input) until all have left for their stations.

While a submarine is in port it may be susceptible to strikes at source. In addition, the submarine building rate creates new submarines as the war progresses. As each new submarine or "turnaround" submarine leaves port, it is given a full load of anti-ASW and anti-shipping torpedoes, and a time at which it must depart from station and return home. Both of these inputs are a function of submarine type. Submarines that are on station at the start of the war are given a full load of both types of torpedoes, but their time remaining on station is determined by a uniform random distribution. The enemy submarine patrol areas are considered to be a constant distance from the submarine port.

While all submarines are considered to transit the same average distance to their patrol area, they can appear in one of four areas along the convoy route, each area involving either good or bad sonar conditions. Each area is looked upon as a "sonar box" and is given a certain percentage of each type of submarine (input). A convoy must pass through the sonar boxes in a

** The terms "detect" or "detection" as used throughout this research contribution refer to the process of sensing a signal and classifying it as due to a target.

fixed sequence, viz., Box 1, first, Box 2, second, etc. Either good or bad sonar conditions may be assigned to each box. If a given submarine is put into a certain box, the submarine derives both its detection and sinking probabilities with respect to the convoy, not only from its type, but also from the sonar conditions assigned to the box. Thus for each decision point in the submarine/convoy engagement, there is in effect a 2 x 3 probability matrix, the rows of which correspond to the two possible sonar conditions, and the columns to the three possible types of submarines.

When the submarine leaves port, it may be required to pass through as many as eight barriers, four constant and four variable, before arriving in the submarine patrol area. Each barrier can be used or not used as desired (input). The constant barriers are of the same type as those described for the convoy, except that in addition to having a probability of being sunk as it transits the barrier, the submarine can also incur a time delay as it goes through or around the barrier. Any time delay so incurred will of course cut down the remaining time the submarine can spend in its patrol area.

When the submarine crosses a variable barrier, it not only has the chance of being sunk itself, but also has the chance of sinking a ship in the barrier. If the submarine does sink a ship in the barrier, the probability of detection for this barrier is assigned a new lower value. The barrier ship sunk is replaced at some later time in the game (an input), and the detection probability of the barrier increased. In this way the variable barrier not only destroys some of the submarines that must transit it, but is itself subjected to possible reductions in effectiveness. Of course, submarines which are detected by the barrier but not sunk suffer time delays which reduce the time that can be spent in the patrol area.

As long as the submarine is at sea, it becomes susceptible to being found and engaged by a HUK group. There are two different probabilities per day at sea that a submarine will encounter a HUK group; one applies when the submarine is in transit to or from its patrol area, and the other when the submarine is in its patrol area waiting for a convoy. Should the submarine be found by a HUK group, the HUK group will attempt to sink the submarine. In turn, the submarine has a chance of sinking one of the HUK ships.

The time of arrival "on station" depends upon the speed of the submarine (which in turn depends upon submarine type) and upon the time delays the submarine undergoes while crossing barriers and fighting HUK groups. The submarine remains on station or in its patrol area (sonar box) until a convoy crosses the "subline." The length of the "subline" or what could better be called the submarine patrol area, is an input.

It is assumed that a convoy crosses the center of the submarine patrol line. When this happens, the first submarine is drawn from Box 1 and placed, using a uniform random distribution, on the subline. Its position on this line, relative to the center of the line, is compared to an athwartship detection range for this type of submarine and for its sonar box. If it is within the detection range, the submarine detects the convoy, otherwise it does not. If it detects the convoy, it will attempt to "close" on this convoy; if it does not detect the convoy, it will be returned to its

sonar box (patrol area).

The first three boxes are played in this manner: the fourth box is played slightly differently. As the convoy approaches the fourth box (fourth patrol area), there is given a probability per month of the war that submarines in this patrol area will contact the convoy. If contact is made, this box will be played as the other three. If, however, this contact is not made, this box will not be played for this convoy. This fourth box might then be used for a submarine wolf pack; for, although a wolf pack has a low probability of intercepting a convoy, given this interception it has a high probability of sinking ships in the convoy.

In general, if a submarine has detected a convoy, it will next try to penetrate the convoy defenses and sink cargo ships. (However, an option is permitted, if desired, by which submarines will try to sink ASW escorts in an anti-ASW escort campaign.) The event that plays the submarine engaging the convoy is Event Subroutine No. 19, "Submarine vs. Convoy." In this event, the submarine must first penetrate the forward screen (or outward screen of the convoy). Given a successful penetration of this screen, it must then penetrate the ASW screen (or innermost convoy screen). When it has successfully penetrated both of these screens, it may then attack cargo ships. When the submarine has sunk one or more ships, either cargo or screen ships, the surviving screen ships form surface attack units (SAU's) which attempt to locate the submarine and attack it. Anti-shipping and anti-ASW torpedoes are used during this attack, and whenever the allowable minimum number of torpedoes of the type needed to continue the attack has been reached, the submarine is required to break off the attack and start its transit for home. ...

After a submarine has either reached its minimum number of torpedoes or has used its allowable time, whichever occurs first, it must return to its home port. On the way home, the submarine must traverse, in reverse order, the same barriers that it crossed on its way out to station. When the submarine arrives at its home port, it will stay there for a certain number of days (input) and then be resupplied and again sail for its patrol area.